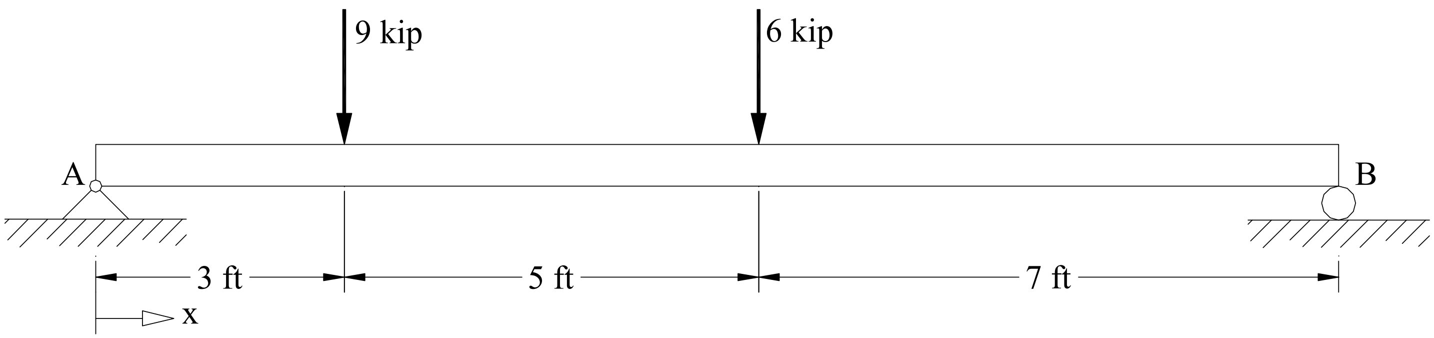
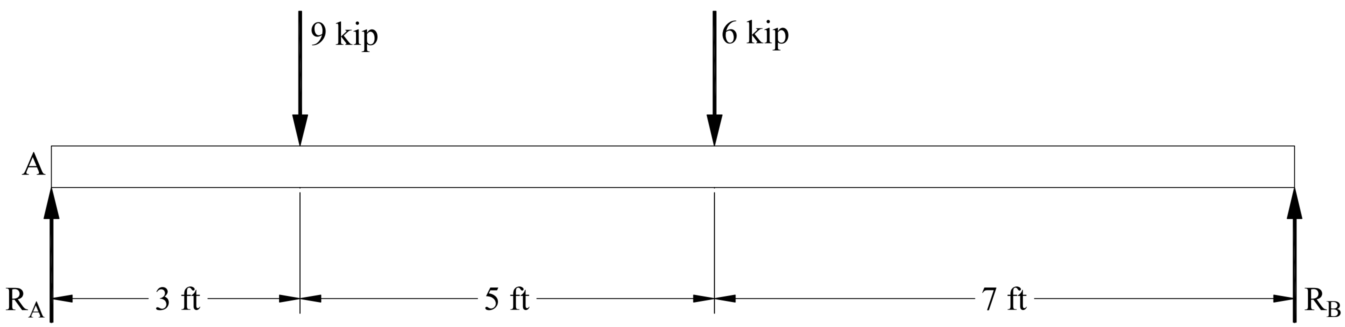
8.2 Shear and Bending-Moment Diagrams: Equation Form

8.2 Shear and Bending-Moment Diagrams: Equation Form Example 1, page 1 of 6

1. Express the shear V and bending moment M as functions of x, the distance from the left end of the beam to an arbitrary point on the beam. Plot V and M vs. x.



(1) Draw a free-body diagram and find the reactions.



$+\uparrow \Sigma F_y = 0$: $R_A - 9\text{ kip} - 6\text{ kip} + R_B = 0$

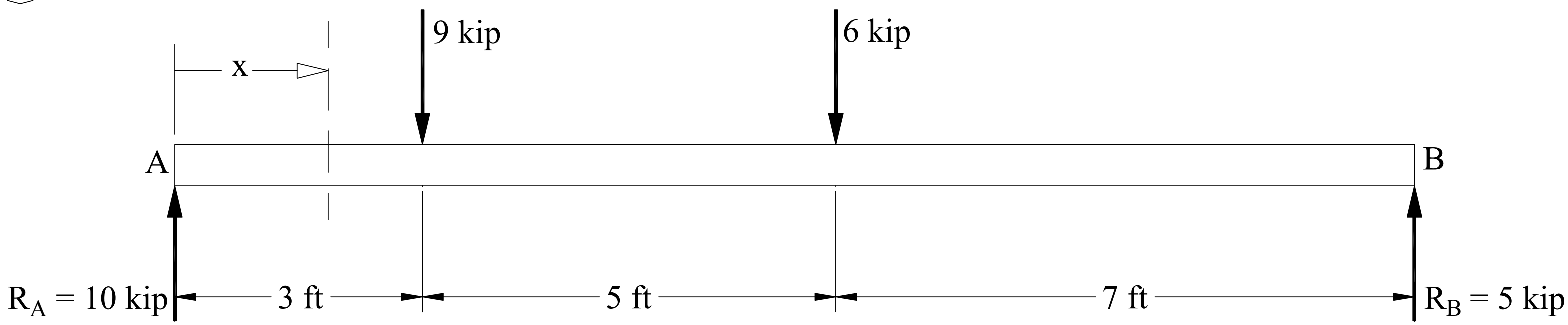
$\circlearrowleft + \Sigma M_A = 0$: $-(9\text{ kip})(3\text{ ft}) - (6\text{ kip})(3\text{ ft} + 5\text{ ft}) + R_B(3\text{ ft} + 5\text{ ft} + 7\text{ ft}) = 0$

Solving gives

$R_A = 10$ kip and $R_B = 5$ kip

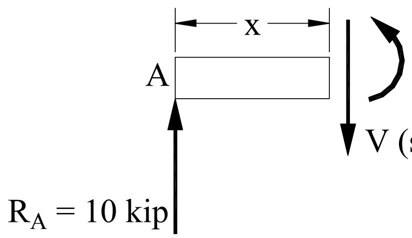
8.2 Shear and Bending-Moment Diagrams: Equation Form Example 1, page 2 of 6

(2) Pass a section through the beam at a point between the left end and the 9-kip force.



$0 < x < 3$ ft

(3) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



M (sign convention: M positive counterclockwise, on *right* end of section)

V (sign convention: V positive down, on *right* end of section)

$+\uparrow \Sigma F_y = 0$: $10 \text{ kip} - V = 0$

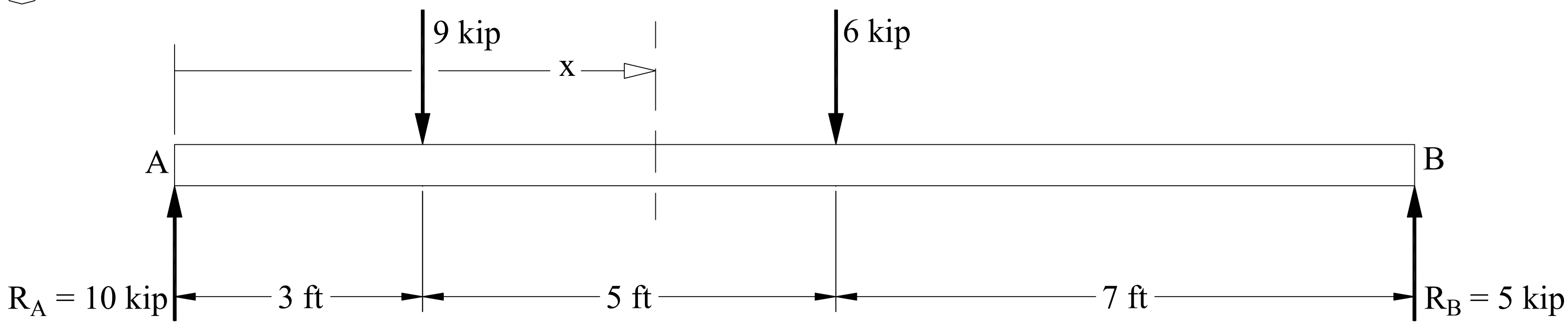
$\circlearrowleft + \Sigma M_x = 0$: $-(10 \text{ kip})x + M = 0$

Solving gives

$V = 10$ kip and $M = 10x$ kip·ft

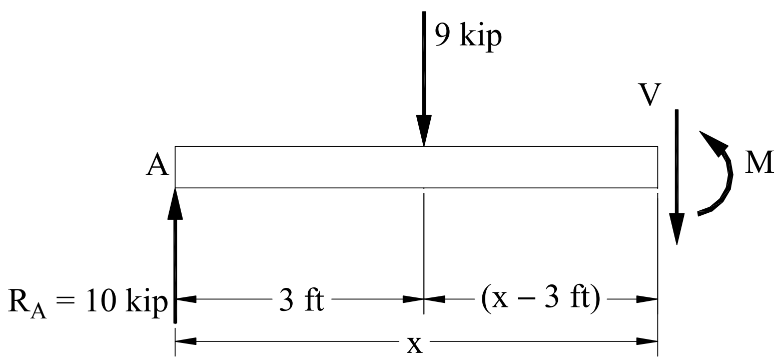
valid for $0 < x < 3$ ft.

(4) Pass a section through the beam at a point between the 9-kip force and the 6-kip force.



3 ft < x < 8 ft

(5) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $10 \text{ kip} - 9 \text{ kip} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $-(10 \text{ kip})x + (9 \text{ kip})(x - 3 \text{ ft}) + M = 0$

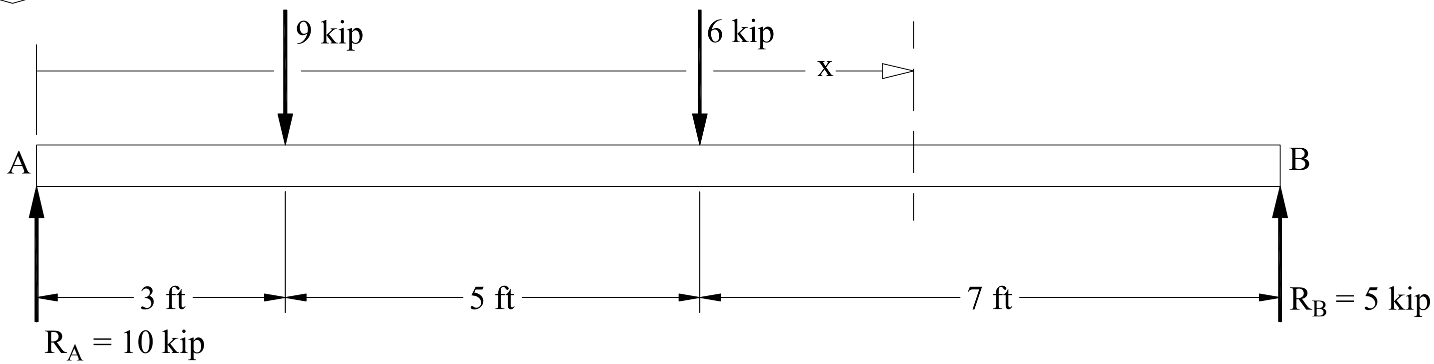
Solving gives

$$V = 1 \text{ kip} \quad (3)$$

$$M = (x + 27) \text{ kip}\cdot\text{ft} \quad (4)$$

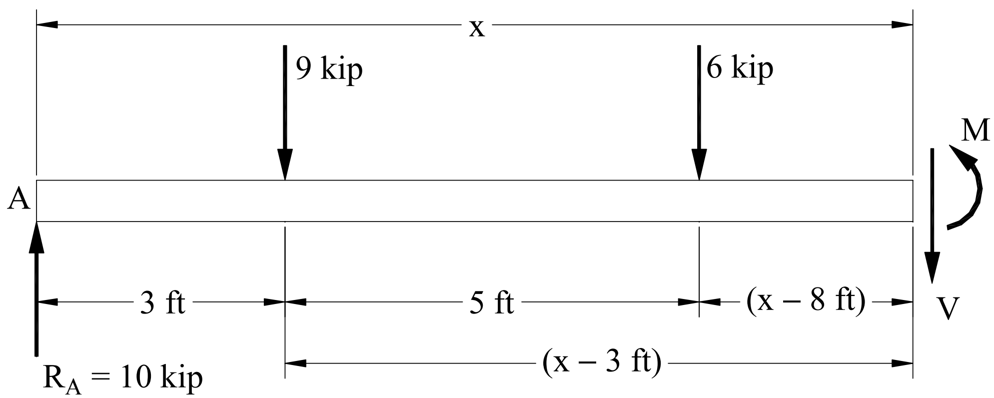
valid for 3 ft < x < 8 ft.

(6) Pass a section through the beam at a point between the 6-kip force and the right end of the beam.



8 ft < x < 15 ft

(7) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $10 \text{ kip} - 9 \text{ kip} - 6 \text{ kip} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $-(10 \text{ kip})x + (9 \text{ kip})(x - 3 \text{ ft})$

$+ (6 \text{ kip})(x - 8 \text{ ft}) + M = 0$

Solving gives

$$V = -5 \text{ kip} \quad (5)$$

$$M = (-5x + 75) \text{ kip}\cdot\text{ft} \quad (6)$$

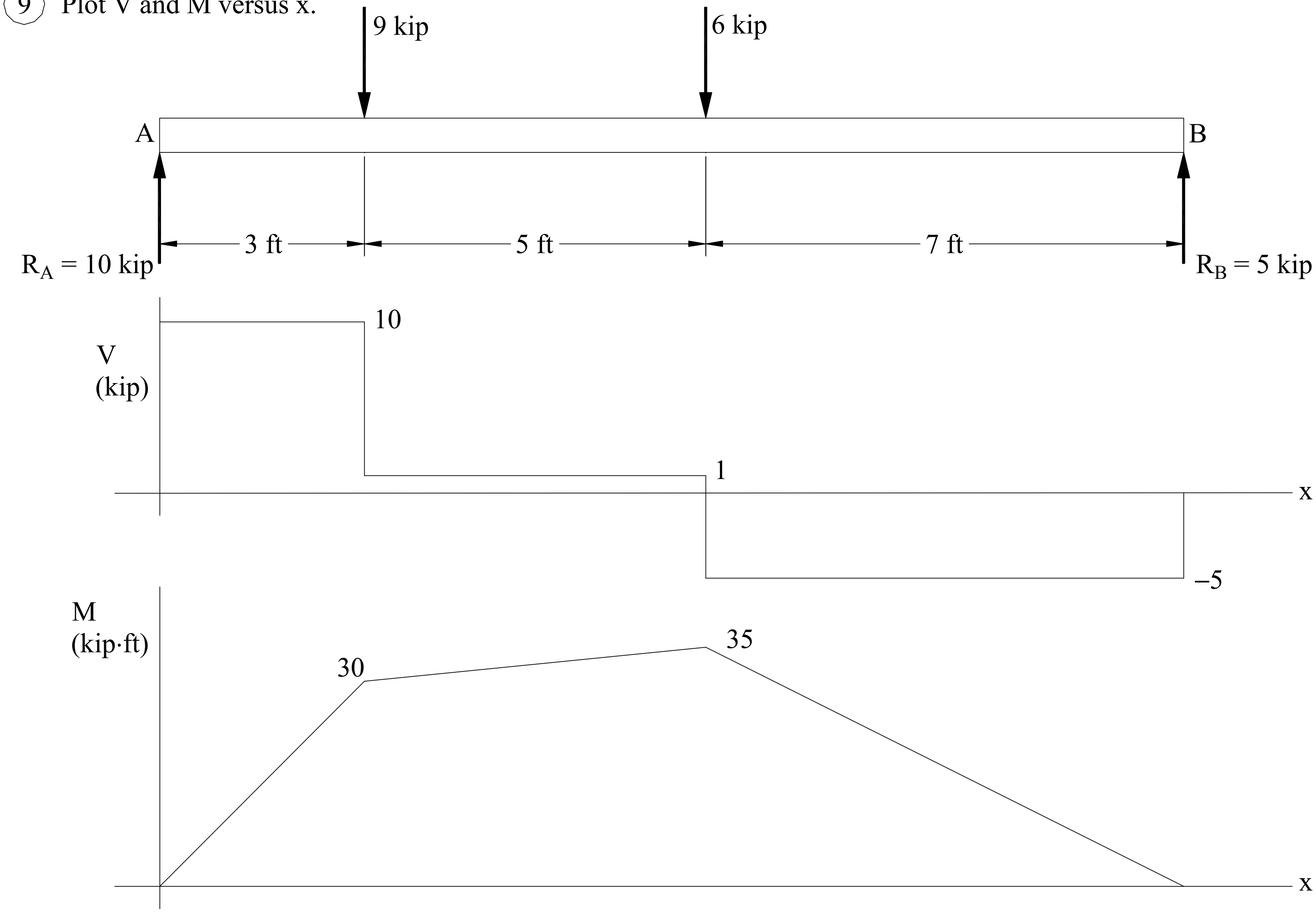
valid for 8 ft < x < 15 ft.

(8) Collect the results from Eqs. 1-6:

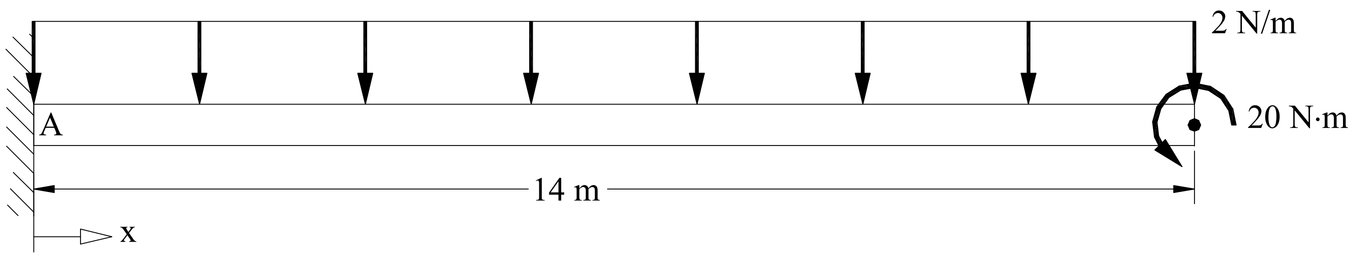
$$\left.\begin{array}{ll} 0 < x < 3\text{ ft} & V = 10\text{ kip} \\ & M = 10x\text{ kip}\cdot\text{ft} \\ 3\text{ ft} < x < 8\text{ ft} & V = 1\text{ kip} \\ & M = (x + 27)\text{ kip}\cdot\text{ft} \\ 8\text{ ft} < x < 15\text{ ft} & V = -5\text{ kip} \\ & M = (-5x + 75)\text{ kip}\cdot\text{ft} \end{array}\right\} \leftarrow \text{Ans.}$$

8.2 Shear and Bending-Moment Diagrams: Equation Form Example 1, page 6 of 6

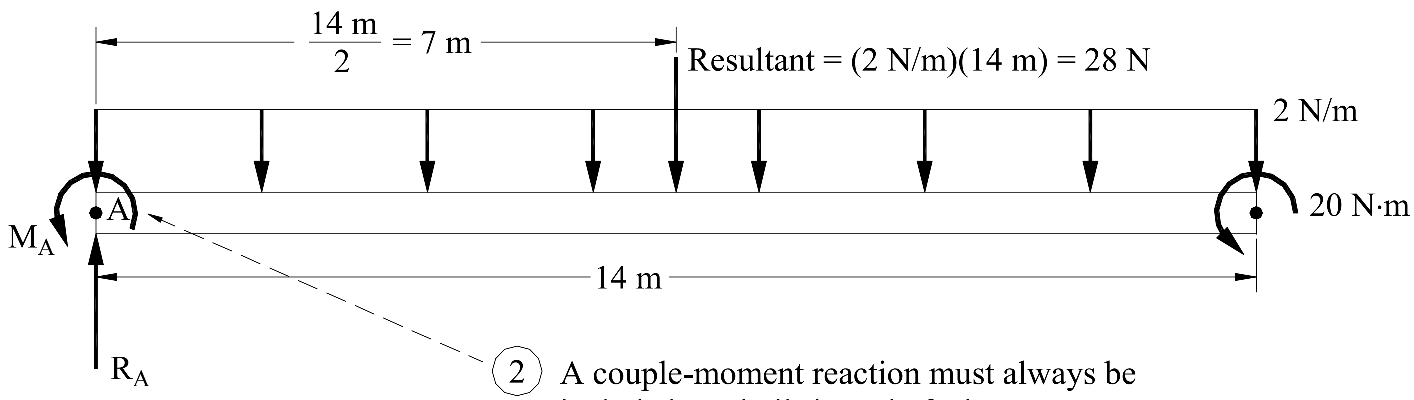
(9) Plot V and M versus x.



2. Express the shear V and bending moment M as functions of x, the distance from the left end of the beam to an arbitrary point on the beam. Plot V and M vs. x.



(1) Draw a free-body diagram and find the reactions.



(2) A couple-moment reaction must always be included at a built-in end of a beam.

$+\uparrow \Sigma F_y = 0$: $R_A - 28 \text{ N} = 0$

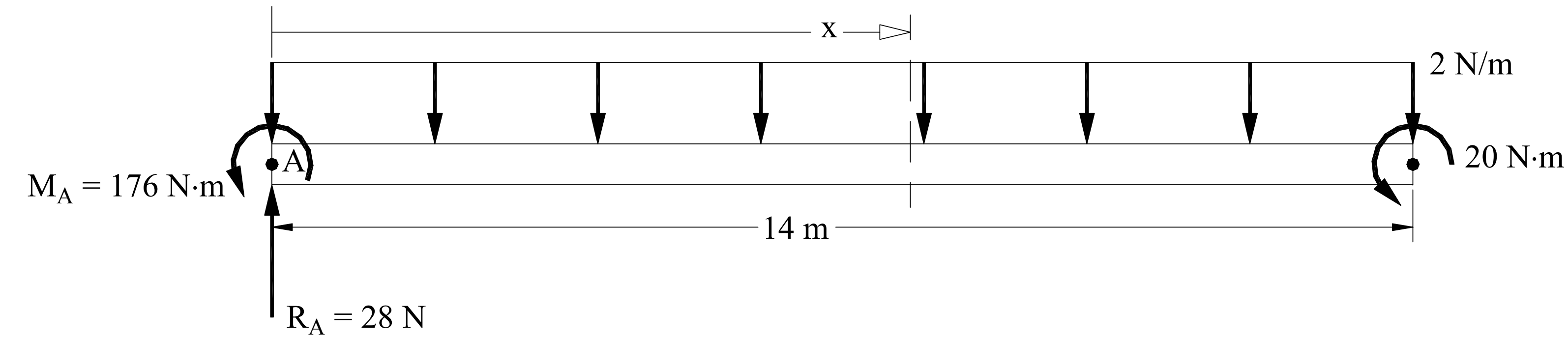
$\circlearrowleft + \Sigma M_A = 0$: $M_A - (28 \text{ N})(7 \text{ m}) + 20 \text{ N·m} = 0$

Solving gives

$R_A = 28$ N and $M_A = 176$ N·m

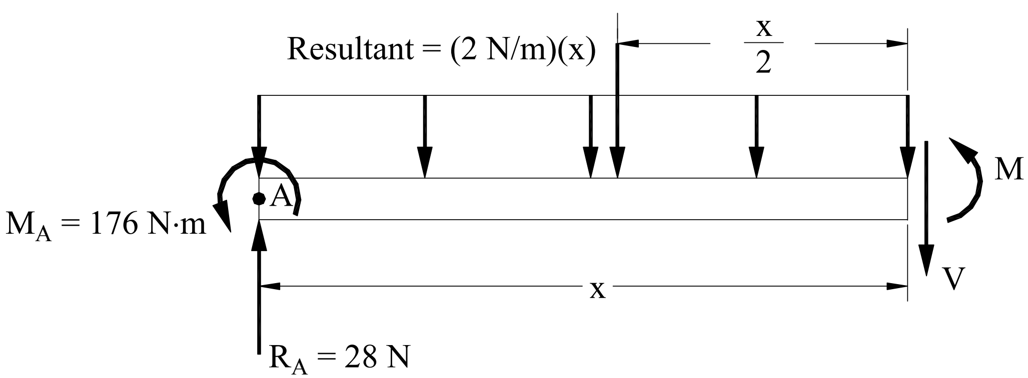
8.2 Shear and Bending-Moment Diagrams: Equation Form Example 2, page 2 of 3

(3) Pass a section through the beam at an arbitrary point (located by x)



$0 < x < 14$ m

(4) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $28\text{ N} - 2x - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $176\text{ N·m} - 28x + \left(\frac{x}{2}\right)(2\text{ N/m})(x) + M = 0$

Solving gives

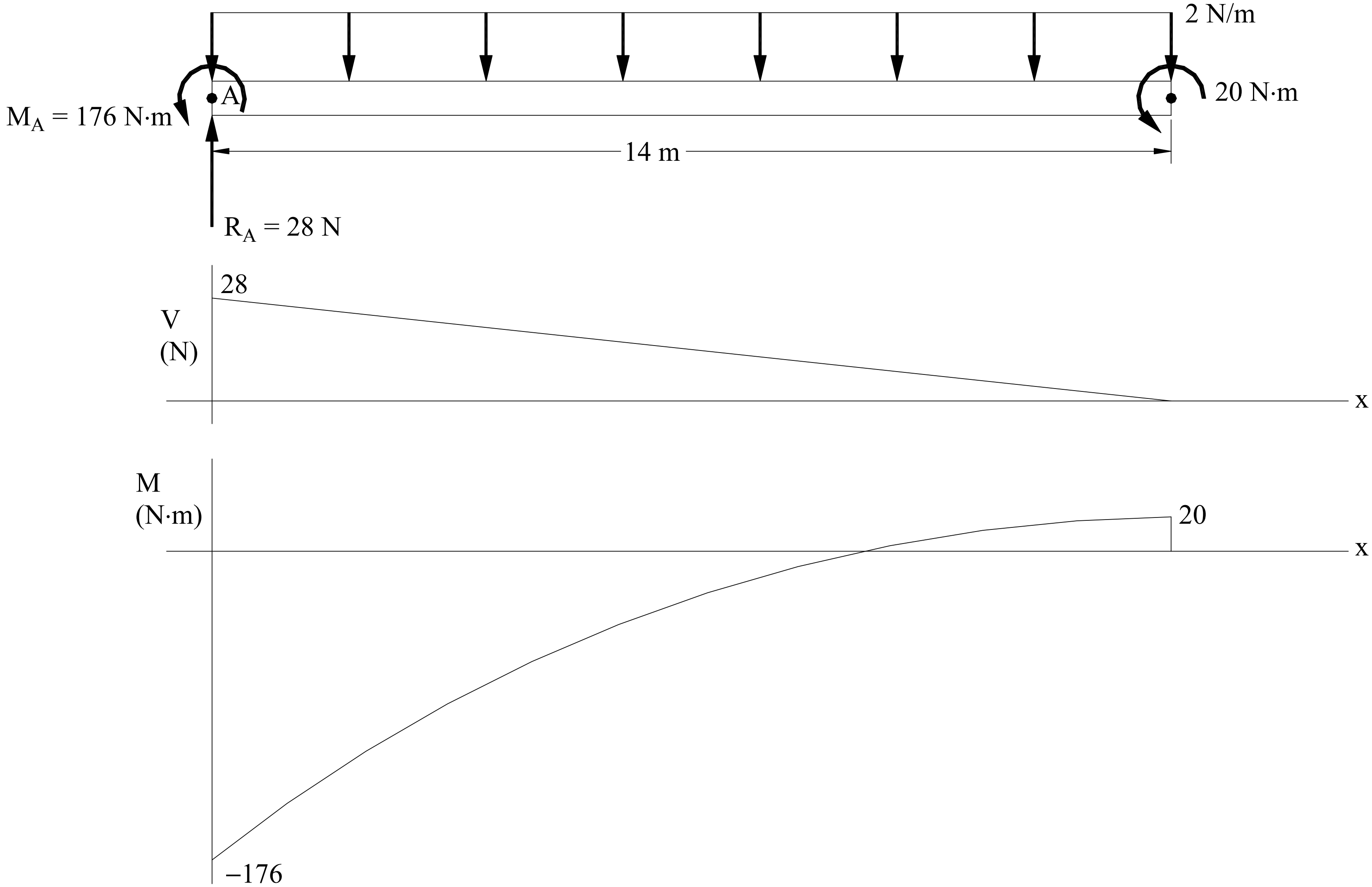
$V = (-2x + 28)\text{ N}$ ←Ans.

$M = (-x^2 + 28x - 176)\text{ N·m}$ ←Ans.

valid for $0 < x < 14$ m.

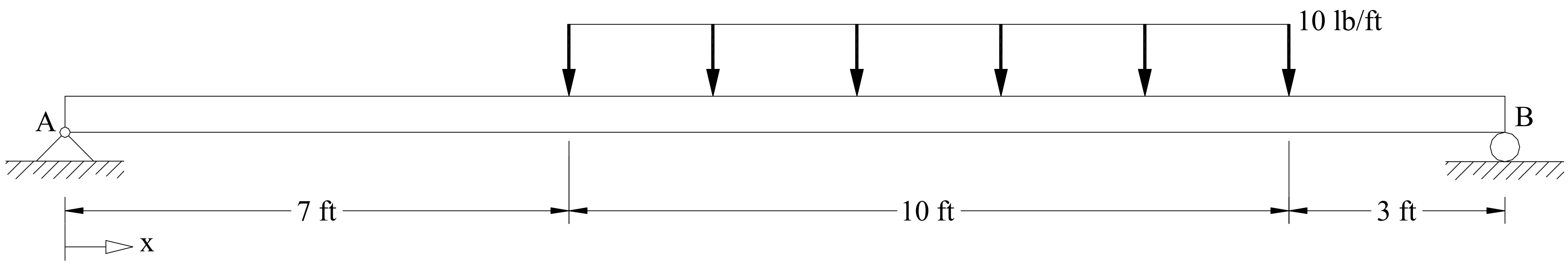
8.2 Shear and Bending-Moment Diagrams: Equation Form Example 2, page 3 of 3

(5) Plot V and M versus x.

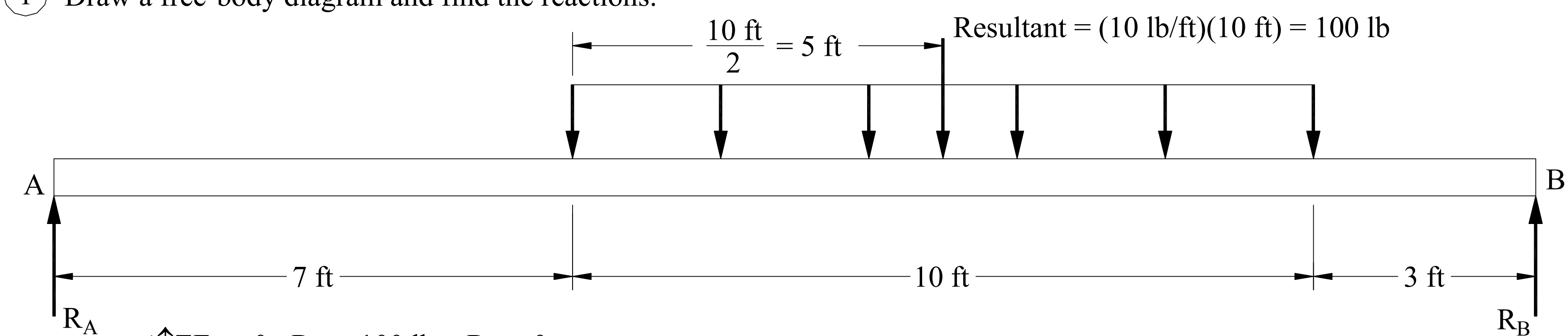


8.2 Shear and Bending-Moment Diagrams: Equation Form Example 3, page 1 of 6

3. Express the shear V and bending moment M as functions of x, the distance from the left end of the beam to an arbitrary point on the beam. Plot V and M vs. x.



(1) Draw a free-body diagram and find the reactions.



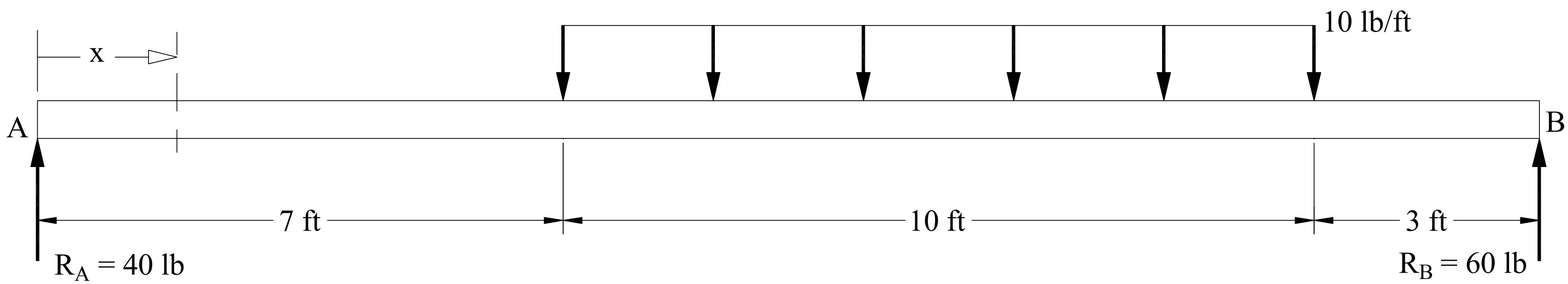
$+\uparrow \Sigma F_y = 0$: $R_A - 100 \text{ lb} + R_B = 0$

$\circlearrowleft + \Sigma M_A = 0$: $-(100 \text{ lb})(7 \text{ ft} + 5 \text{ ft}) + R_B(7 \text{ ft} + 10 \text{ ft} + 3 \text{ ft}) = 0$

Solving gives

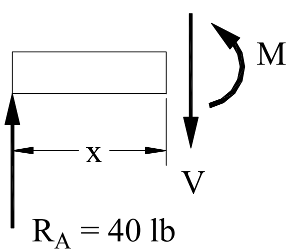
$R_A = 40$ lb and $R_B = 60$ lb

(2) Pass a section through the beam at a point between the left end of the beam and the beginning of the distributed load.



$0 < x < 7$ ft

(3) Draw a free-body diagram and find the reactions.



$+\uparrow \Sigma F_y = 0$: $40 \text{ lb} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $-(40 \text{ lb})x + M = 0$

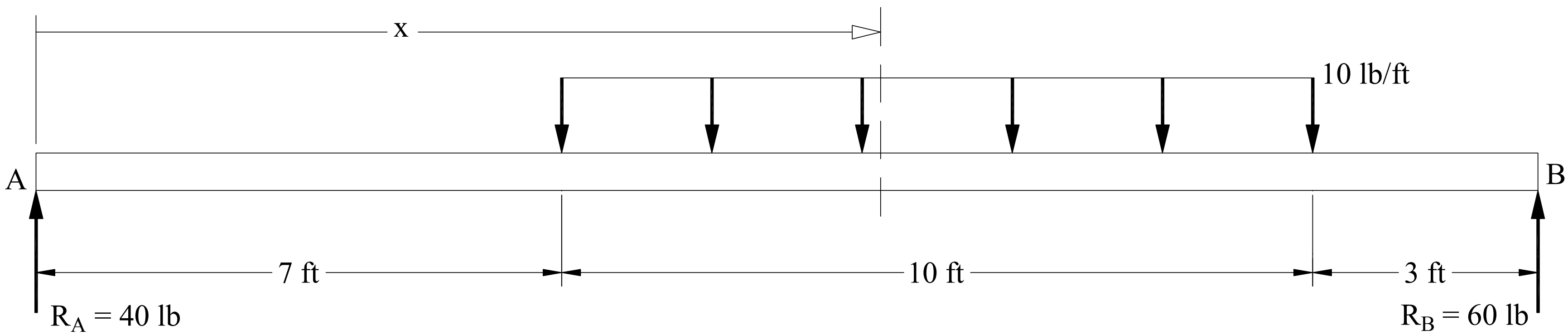
Solving gives

$$V = 40 \text{ lb} \quad (1)$$

$$M = (40x) \text{ lb·ft} \quad (2)$$

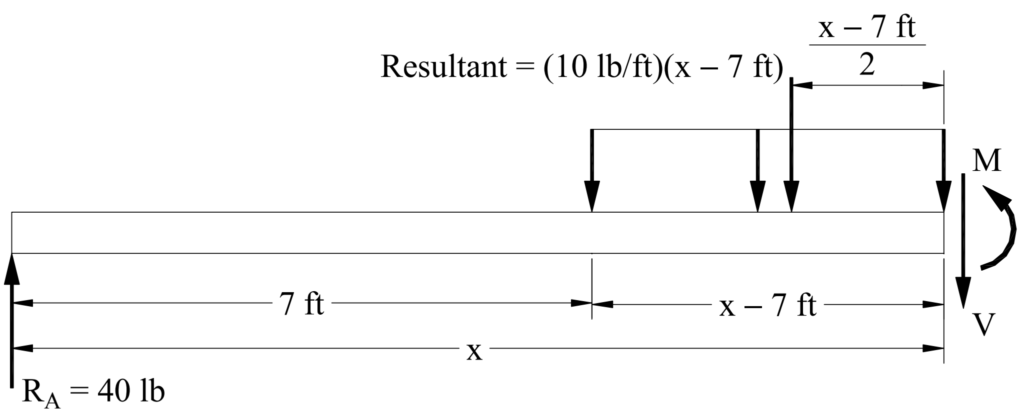
valid for $0 < x < 7$ ft.

(4) Pass a section through the beam at a point between the beginning and end of the distributed load.



$7\text{ ft} < x < 17\text{ ft}$

(5) Draw a free-body diagram of the portion of the beam to the left of the section and solve for V and M at the section.



$$+\uparrow \Sigma F_y = 0: \; 40\text{ lb} - (10\text{ lb/ft})(x - 7\text{ ft}) - V = 0$$

$$\circlearrowleft + \Sigma M_x = 0: \; -(40\text{ lb})x + [(10\text{ lb/ft})(x - 7\text{ ft})] \times \left(\frac{x - 7\text{ ft}}{2}\right) + M = 0$$

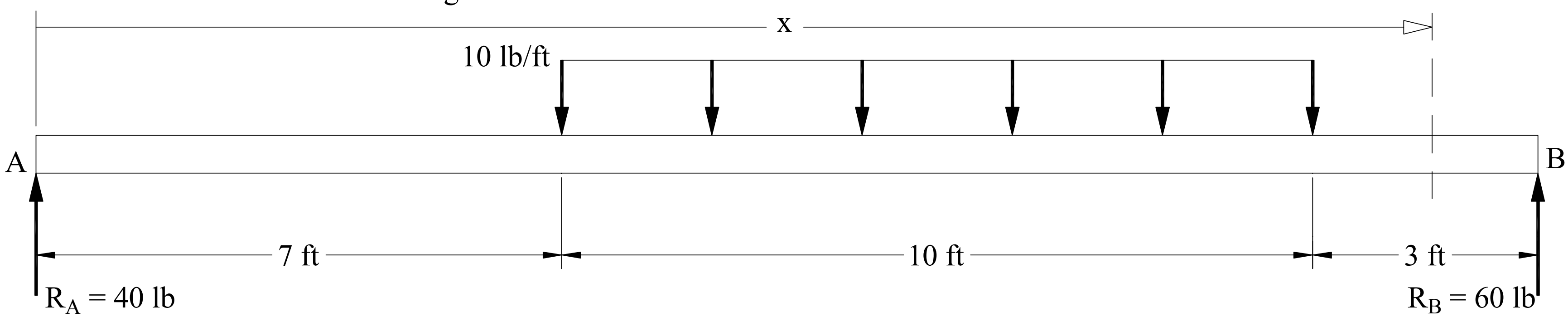
Solving gives

$$V = (-10x + 110)\text{ lb} \qquad (3)$$

$$M = (-5x^2 + 110x - 245)\text{ lb·ft} \qquad (4)$$

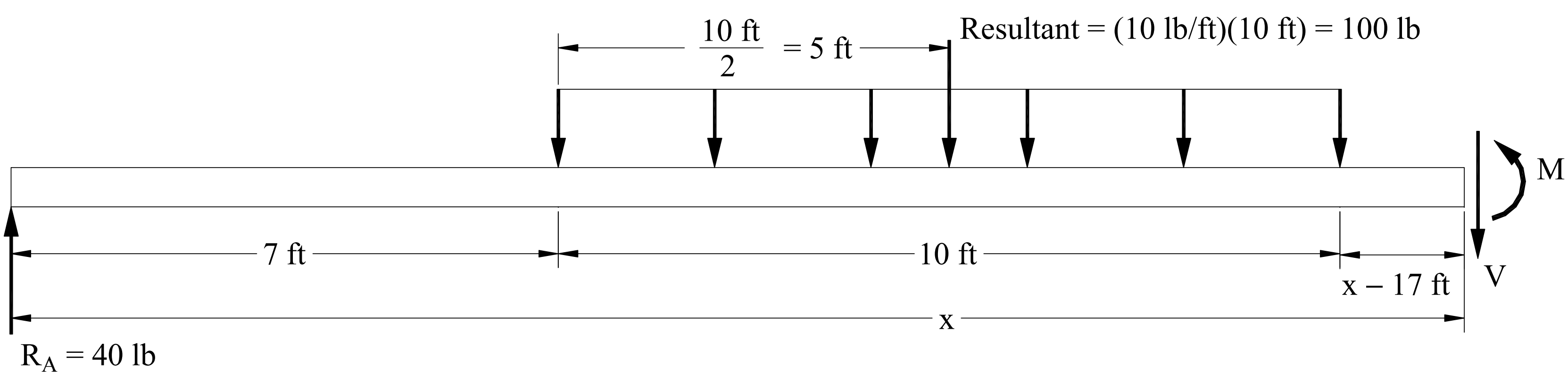
valid for $7\text{ ft} < x < 17\text{ ft}$.

(6) Pass a section through the beam at a point between the right end of the distributed load and the right end of the beam.



17 ft < x < 20 ft

(7) Draw a free-body diagram of the portion of the beam to the left of the section and solve for V and M at the section.



(8) $+\uparrow\Sigma F_y = 0$: $40 \text{ lb} - 100 \text{ lb} - V = 0$

$\circlearrowleft + \Sigma M_A = 0$: $-(40 \text{ lb})x + (100 \text{ lb})[(x - 17 \text{ ft}) + 5 \text{ ft}] + M = 0$

Solving gives

$$V = -60 \text{ lb} \qquad (5)$$

$$M = (-60x + 1200) \text{ lb}\cdot\text{ft} \qquad (6)$$

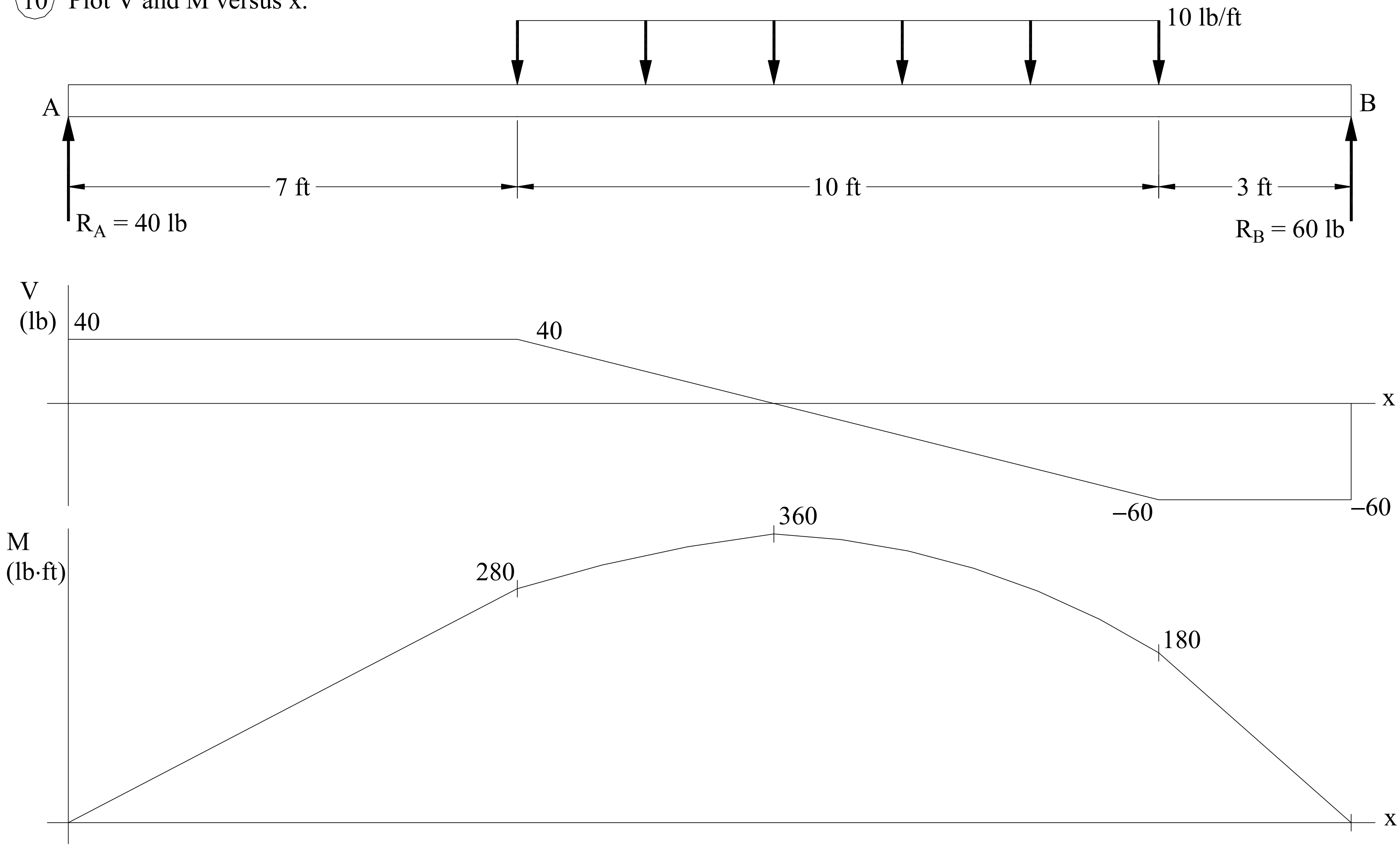
valid for $17 \text{ ft} < x < 20 \text{ ft}$.

(9) Collect the results from Eqs. 1-6:

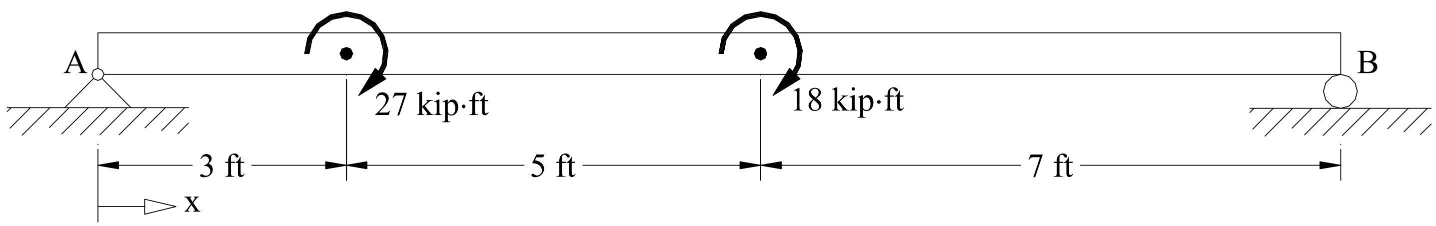
$$\left.\begin{array}{ll} 0 < x < 7 \text{ ft} & V = 40 \text{ lb} \\ & M = 40x \text{ lb}\cdot\text{ft} \\ 7 \text{ ft} < x < 17 \text{ ft} & V = (-10x + 110) \text{ lb} \\ & M = (-5x^2 + 110x - 245) \text{ lb}\cdot\text{ft} \\ 17 \text{ ft} < x < 20 \text{ ft} & V = -60 \text{ lb} \\ & M = (-60x + 1200) \text{ lb}\cdot\text{ft} \end{array}\right\} \leftarrow \text{Ans.}$$

8.2 Shear and Bending-Moment Diagrams: Equation Form Example 3, page 6 of 6

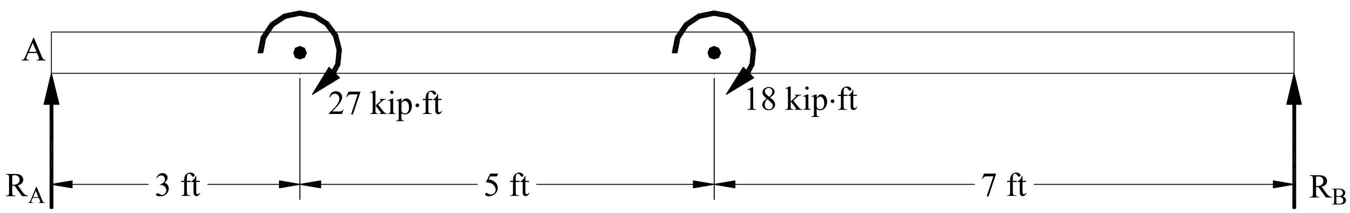
(10) Plot V and M versus x.



4. Express the shear V and bending moment M as functions of x, the distance from the left end of the beam to an arbitrary point on the beam. Plot V and M vs. x.



(1) Draw a free-body diagram and find the reactions.



$+\uparrow \Sigma F_y = 0$: $R_A + R_B = 0$

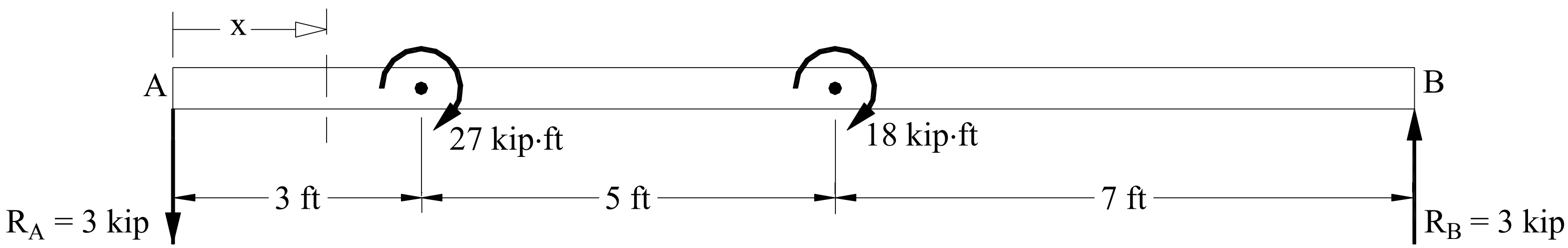
$\circlearrowleft + \Sigma M_A = 0$: $R_B(3 \text{ ft} + 5 \text{ ft} + 7 \text{ ft}) - 27 \text{ kip·ft} - 18 \text{ kip·ft} = 0$

Solving gives

$R_A = -3 \text{ kip} = 3 \text{ kip} \downarrow$

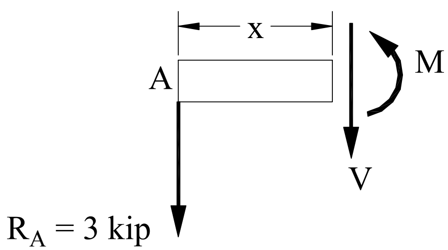
$R_B = 3 \text{ kip}$

(2) Pass a section through the beam at a point between the left end and the 27 kip·ft moment couple.



$0 < x < 3$ ft

(3) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $-3 \text{ kip} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $(3 \text{ kip})x + M = 0$

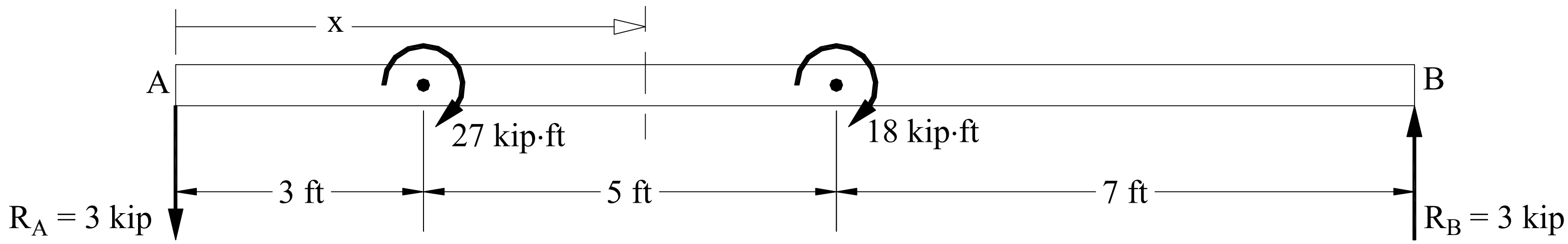
Solving gives

$$V = -3 \text{ kip} \quad (1)$$

$$M = -3x \text{ kip·ft} \quad (2)$$

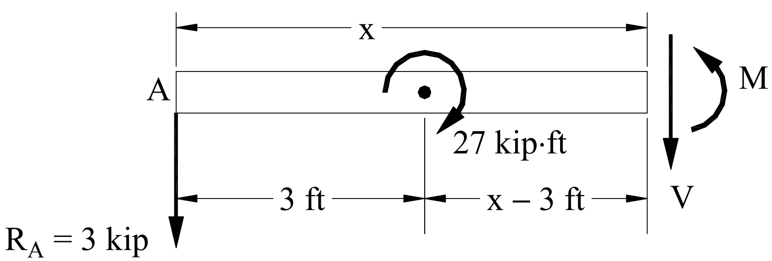
valid for $0 < x < 3$ ft.

(4) Pass a section through the beam at a point between the 27 kip·ft and 18 kip·ft moment couples.



$3 \text{ ft} < x < 8 \text{ ft}$

(5) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $-3 \text{ kip} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $(3 \text{ kip})x - 27 \text{ kip·ft} + M = 0$

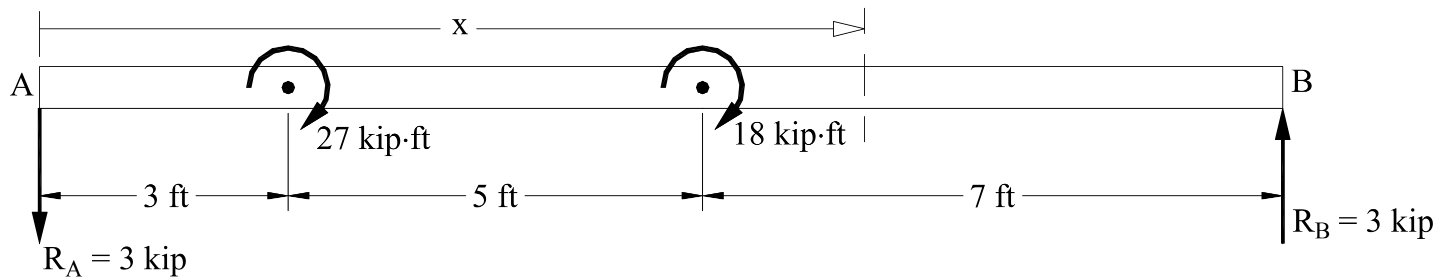
Solving gives

$$V = -3 \text{ kip} \qquad (3)$$

$$M = (-3x + 27) \text{ kip·ft} \qquad (4)$$

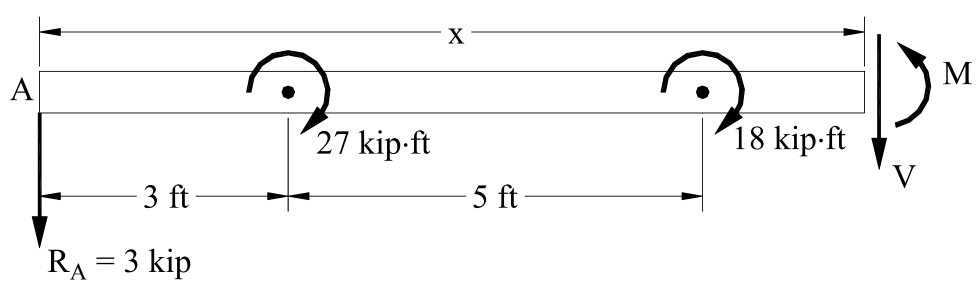
valid for $3 \text{ ft} < x < 8 \text{ ft}$.

(6) Pass a section through the beam at a point between the 18 kip·ft moment couple and the right end of the beam.



3 ft < x < 8 ft

(7) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $-3 \text{ kip} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $(3 \text{ kip})x - 27 \text{ kip·ft} - 18 \text{ kip·ft} + M = 0$

Solving gives

$$V = -3 \text{ kip} \qquad (5)$$

$$M = (-3x + 45) \text{ kip·ft} \qquad (6)$$

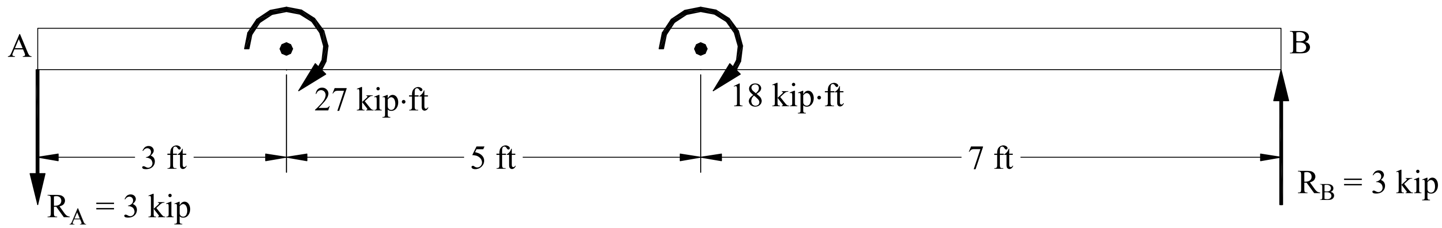
valid for 8 ft < x < 15 ft.

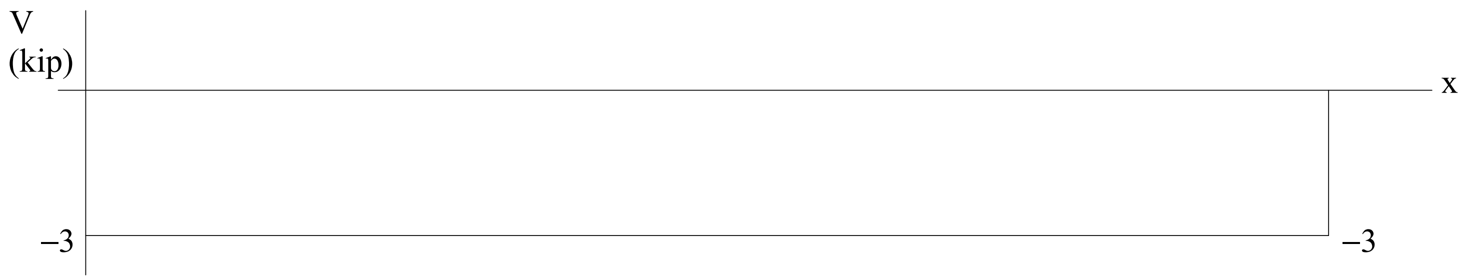
(8) Collect the results from Eqs. 1-6:

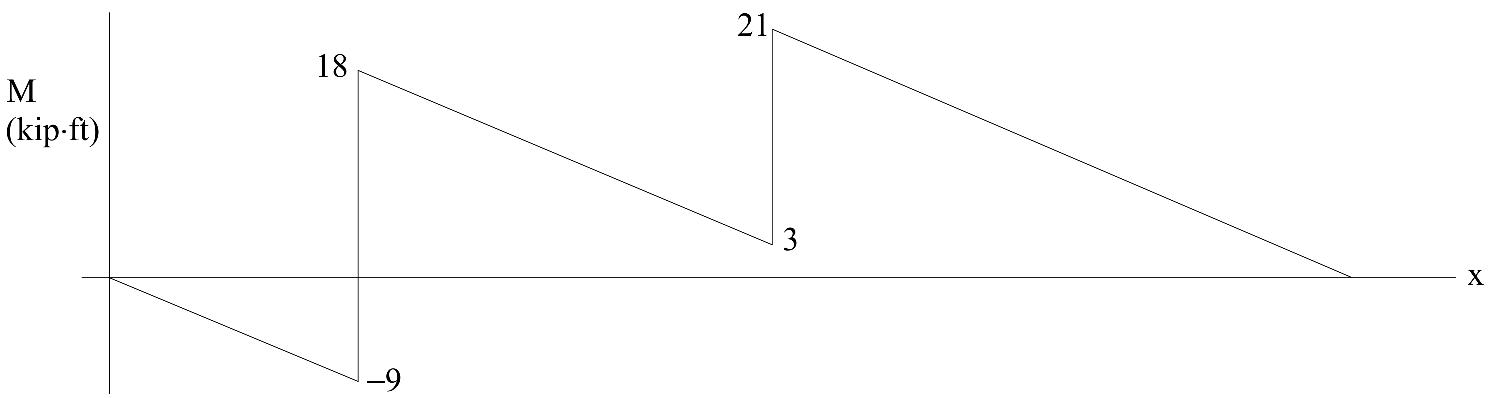
$0 < x < 3$ ft	$V = -3$ kip	
	$M = -3x$ kip·ft	
3 ft $< x < 8$ ft	$V = -3$ kip	←Ans.
	$M = (-3x + 27)$ kip·ft	
8 ft $< x < 15$ ft	$V = -3$ kip	
	$M = (-3x + 45)$ kip·ft	

8.2 Shear and Bending-Moment Diagrams: Equation Form Example 4, page 6 of 6

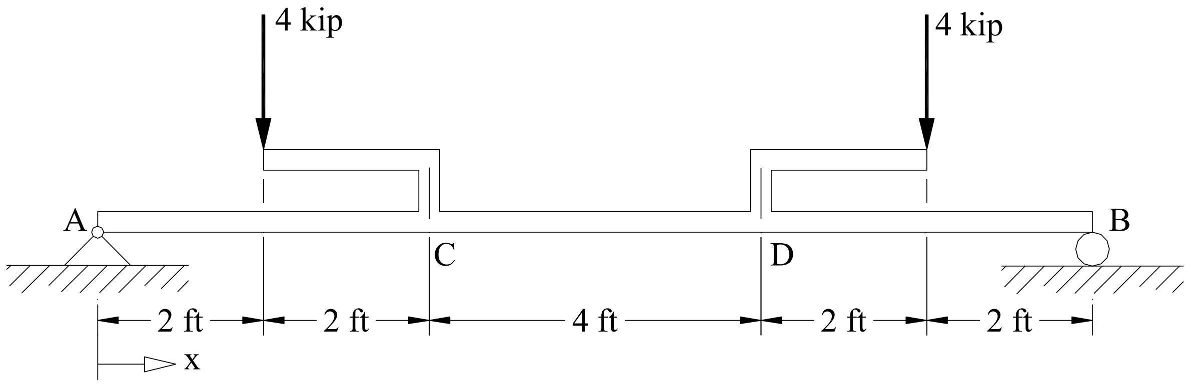
(9) Plot V and M versus x.



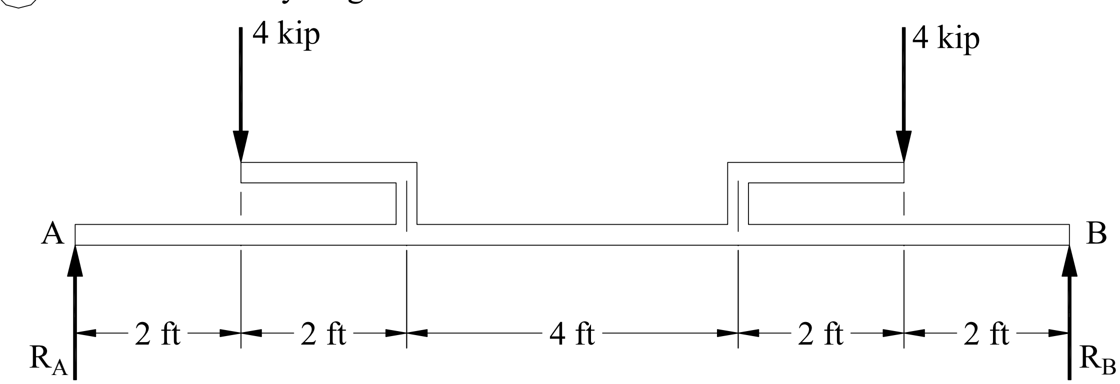




5. Express the shear V and bending moment M in the horizontal portion ACDB of the beam as functions of x, the distance from the left end of the beam to an arbitrary point on the beam. Plot V and M versus x.



(1) Draw a free-body diagram and find the reactions.



$+\uparrow \Sigma F_y = 0$: $R_A - 4 \text{ kip} - 4 \text{ kip} + R_B = 0$

$\circlearrowleft + \Sigma M_A = 0$: $-(4 \text{ kip})(2 \text{ ft}) - (4 \text{ kip})(10 \text{ ft})$

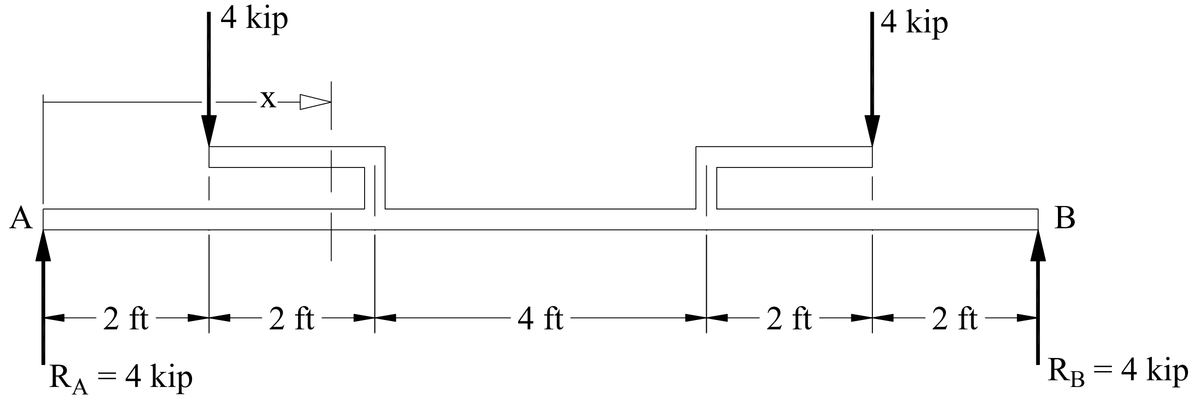
$+ R_B(12 \text{ ft}) = 0$

Solving gives

$R_A = 4$ kip

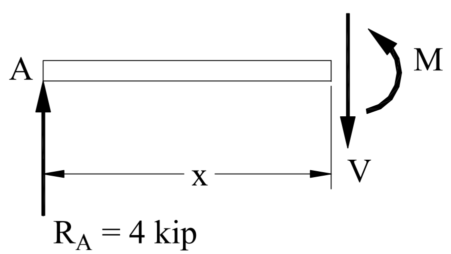
$R_B = 4$ kip

(2) Pass a section through the beam at a point between the left end and the attachment point for the first arm.



$0 < x < 4$ ft

(3) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section. Note carefully that the 4-kip force on the left arm does *not* act on this free body.



$+\uparrow \Sigma F_y = 0$: 4 kip $- V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $-(4 \text{ kip})x + M = 0$

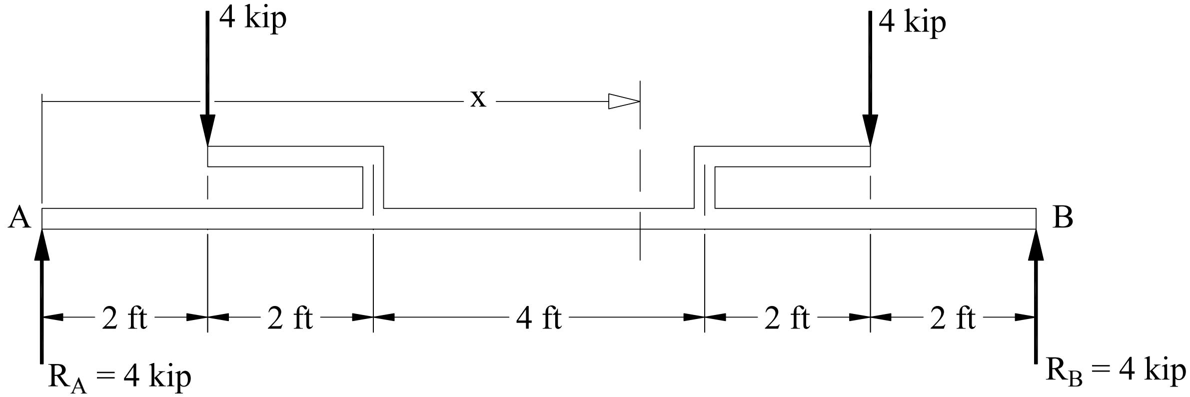
Solving gives

$$V = 4 \text{ kip} \qquad (1)$$

$$M = 4x \text{ kip}\cdot\text{ft} \qquad (2)$$

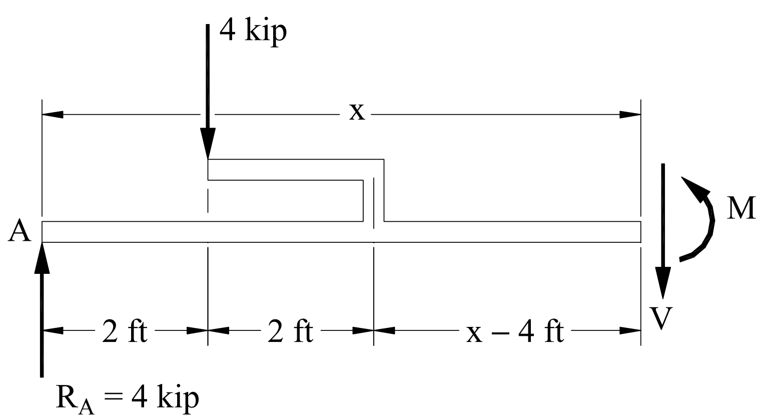
valid for $0 < x < 4$ ft.

(4) Pass a section through the beam at a point between the attachment points of the two arms.



$4 \text{ ft} < x < 8 \text{ ft}$

(5) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $4 \text{ kip} - 4 \text{ kip} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $-(4 \text{ kip})x + (4 \text{ kip})(x - 4 \text{ ft} + 2 \text{ ft}) + M = 0$

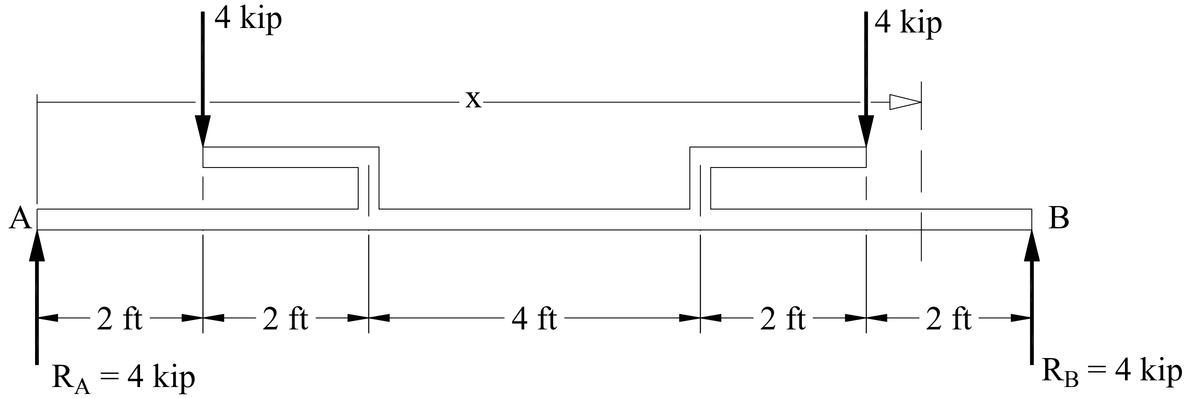
Solving gives

$$V = 0 \qquad (3)$$

$$M = 8 \text{ kip}\cdot\text{ft} \qquad (4)$$

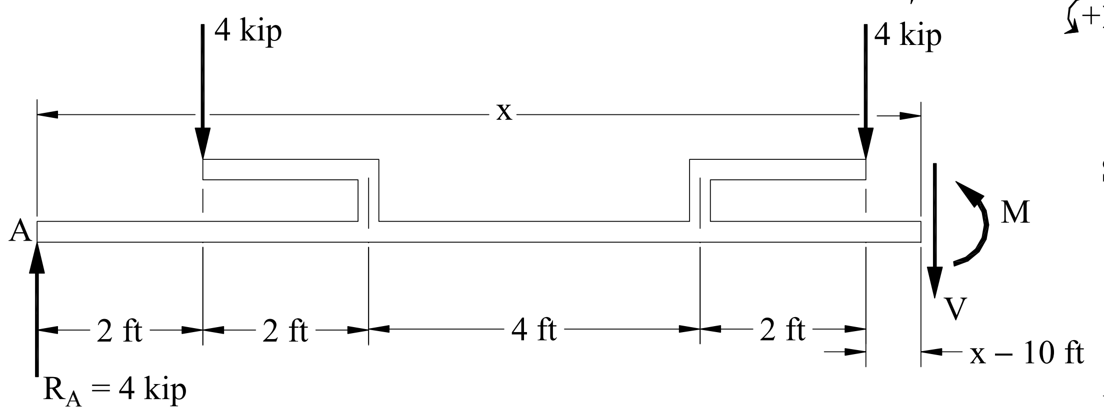
valid for $4 \text{ ft} < x < 8 \text{ ft}$.

(6) Pass a section through the beam at a point between the point of attachment of the right arm and the right end of the beam.



$8 \text{ ft} < x < 12 \text{ ft}$

(7) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



(8) Note that the 4-kip force on the right arm acts on the free body.

$+\uparrow \Sigma F_y = 0$: $4 \text{ kip} - 4 \text{ kip} - 4 \text{ kip} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $-(4 \text{ kip})x + (4 \text{ kip})(x - 2 \text{ ft})$

$+ (4 \text{ kip})(x - 10 \text{ ft}) + M = 0$

Solving gives

$$V = -4 \text{ kip} \quad (5)$$

$$M = (-4x + 48) \text{ kip}\cdot\text{ft} \quad (6)$$

valid for $8 \text{ ft} < x < 12 \text{ ft}$.

(9) Collect the results from Eqs. 1-6:

$0 < x < 4$ ft $V = 4$ kip

$M = 4x$ kip·ft

4 ft $< x < 8$ ft $V = 0$ kip

$M = 8$ kip·ft

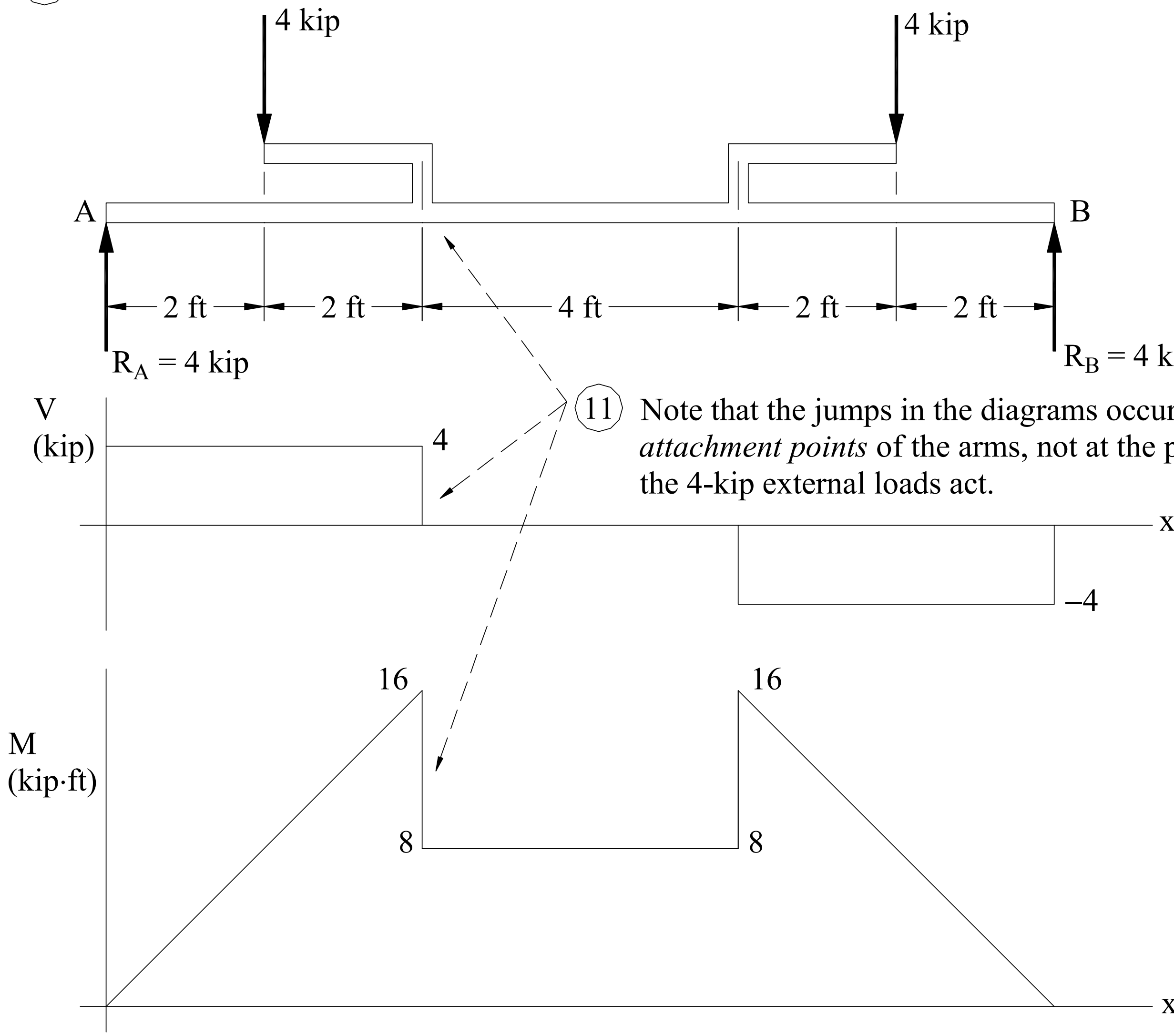
8 ft $< x < 12$ ft $V = -4$ kip

$M = -4(x - 12)$ kip·ft

←Ans.

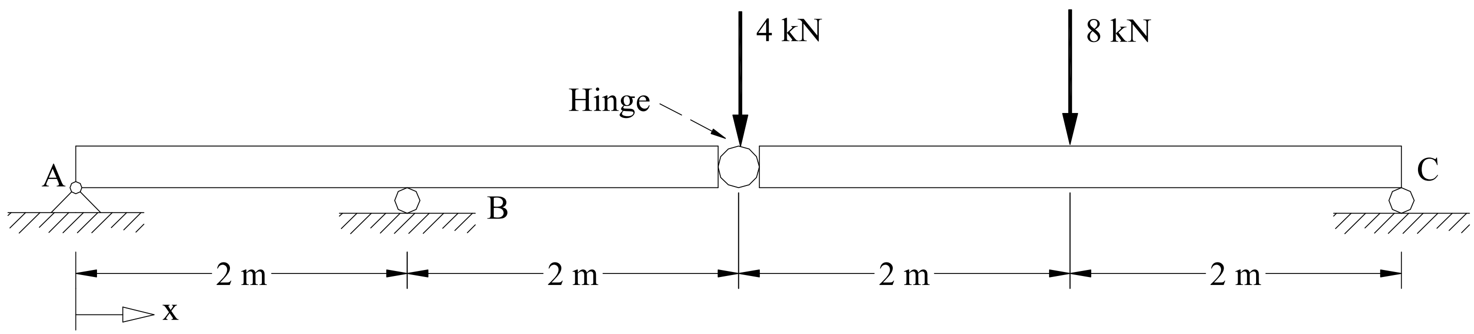
8.2 Shear and Bending-Moment Diagrams: Equation Form Example 5, page 6 of 6

(10) Plot V and M versus x.

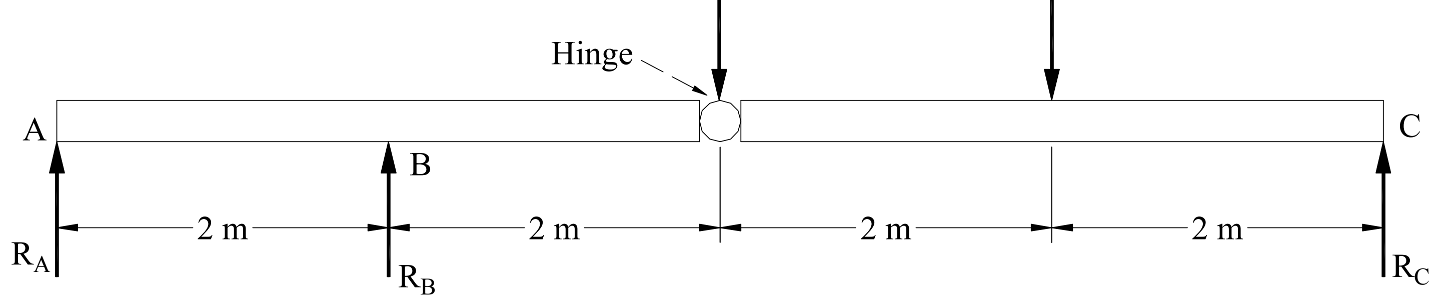


(11) Note that the jumps in the diagrams occur at the *attachment points* of the arms, not at the points where the 4-kip external loads act.

6. Express the shear V and bending moment M as functions of x, the distance from the left end of the beam to an arbitrary point on the beam. Plot V and M vs. x.



(1) Draw a free-body diagram and find the reactions.



$$+\uparrow\Sigma F_y = 0: \; R_A + R_B - 4\text{ kN} - 8\text{ kN} + R_C = 0 \qquad (1)$$

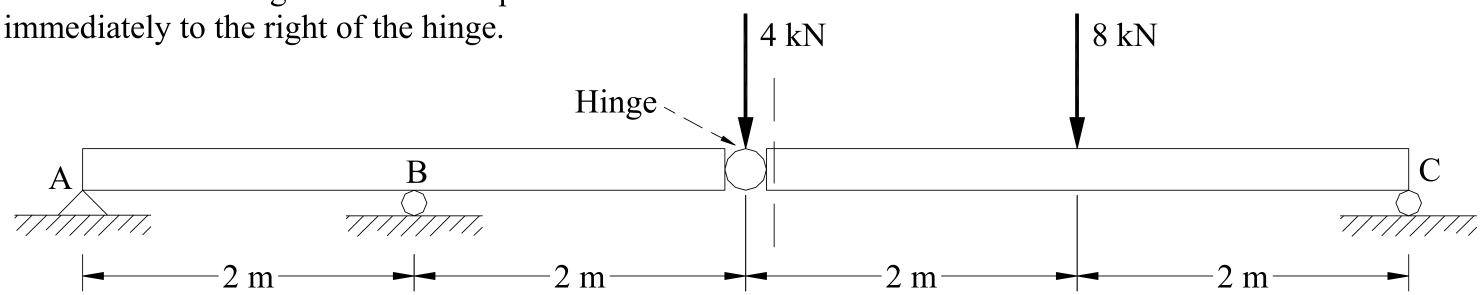
$$\circlearrowleft+\Sigma M_A = 0: \; R_B(2\text{ m}) - (4\text{ kN})(2\text{ m} + 2\text{ m})$$

$$- (8\text{ kN})(2\text{ m} + 2\text{ m} + 2\text{ m})$$

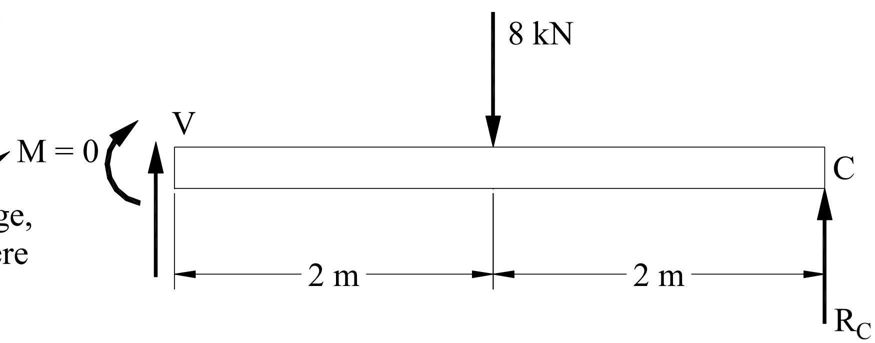
$$+ R_C(2\text{ m} + 2\text{ m} + 2\text{ m} + 2\text{ m}) = 0 \qquad (2)$$

(2) Two equations but three unknowns. An additional equation is needed.

(3) Pass a section through the beam at a point immediately to the right of the hinge.



(4) Draw a free-body diagram of the portion of the beam to the right of the section.



(5) Because the section is next to a hinge, the moment is known to be zero there (that's what we mean by a "hinge").

(6) Write the equilibrium equation for the sum of moments about the hinge.

$$\circlearrowleft + \Sigma M_{hinge} = 0: \; -(8 \text{ kN})(2 \text{ m}) + R_c(2 \text{ m} + 2 \text{ m}) = 0 \qquad (3)$$

(7) Note that we don't use the equation $\Sigma F_y = 0$, because this equation would introduce an additional unknown, the shear V at the hinge.

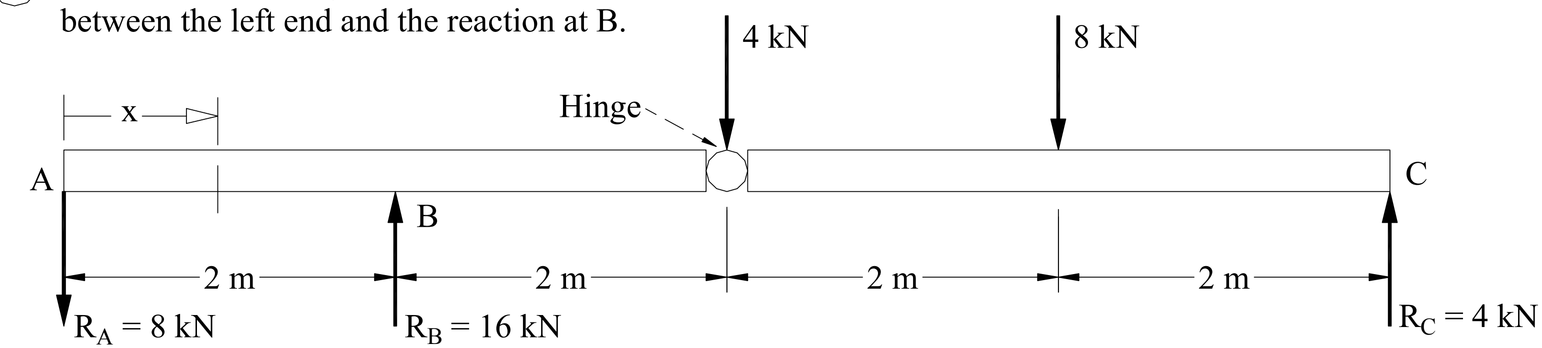
(8) Solving Eqs. 1-3 gives

$R_A = -8 \text{ kN} = 8 \text{ kN} \downarrow$

$R_B = 16 \text{ kN} \uparrow$

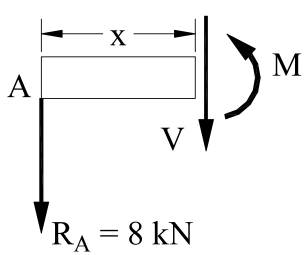
$R_C = 4 \text{ kN} \uparrow$

(9) Pass a section through the beam at a point between the left end and the reaction at B.



$0 < x < 2 \text{ m}$

(10) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0: \; -8 \text{ kN} - V = 0$

$\circlearrowleft + \Sigma M_x = 0: \; (8 \text{ kN})x + M = 0$

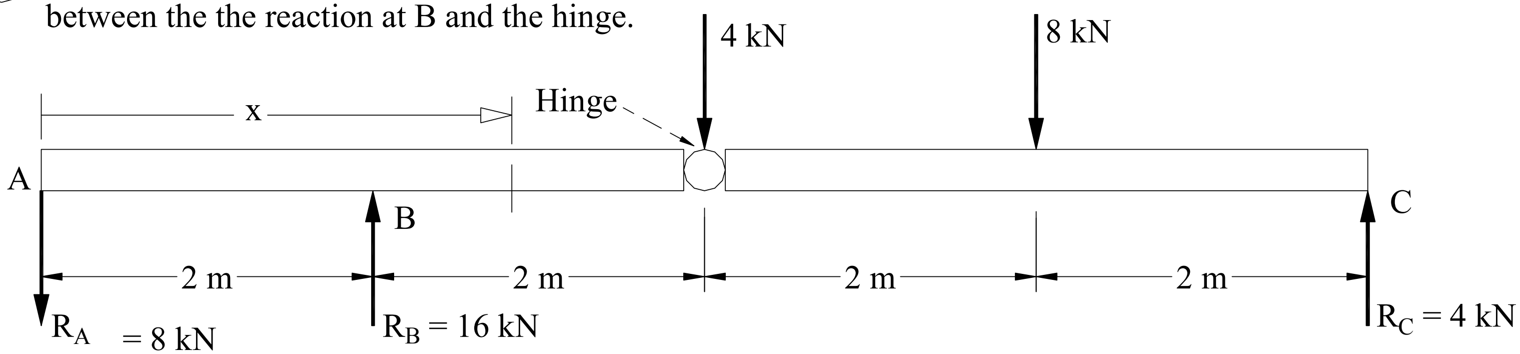
Solving gives

$$V = -8 \text{ kN} \qquad (4)$$

$$M = (-8x) \text{ kN·m} \qquad (5)$$

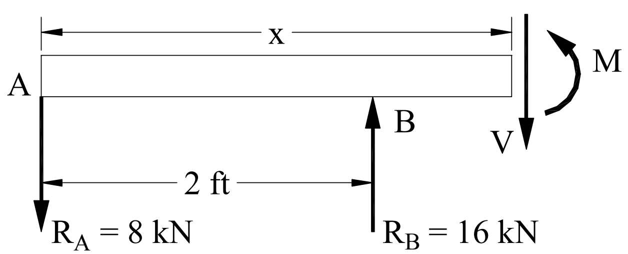
valid for $0 < x < 2$ m.

(11) Pass a section through the beam at a point between the the reaction at B and the hinge.



2 m < x < 4 m

(12) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow\Sigma F_y = 0$: $-8\text{ kN} + 16\text{ kN} - V = 0$

$\circlearrowleft+\Sigma M_x = 0$: $(8\text{ kN})x - (16\text{ kN})(x - 2\text{ m}) + M = 0$

Solving gives

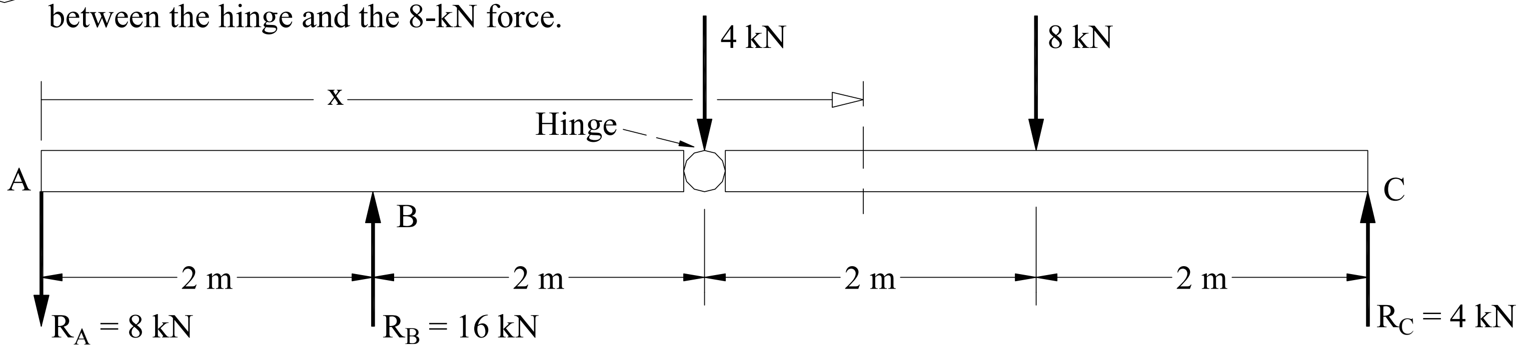
$$V = 8\text{ kN} \quad (6)$$

$$M = (8x - 32)\text{ kN·m} \quad (7)$$

valid for 2 m < x < 4 m.

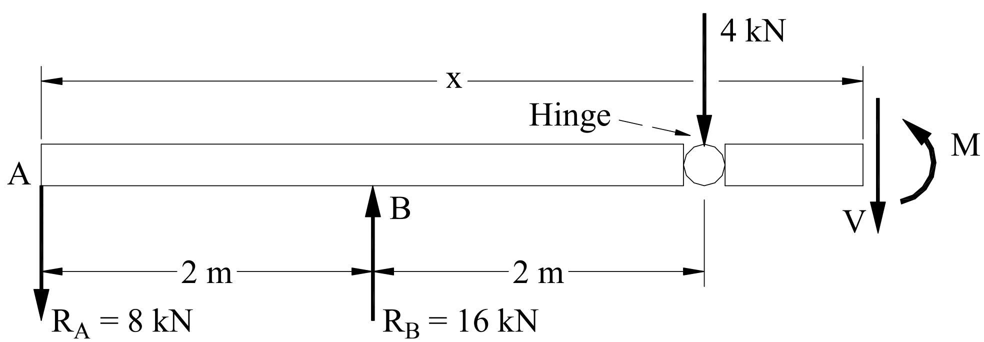
8.2 Shear and Bending-Moment Diagrams: Equation Form Example 6, page 5 of 8

(13) Pass a section through the beam at a point between the hinge and the 8-kN force.



$4 \text{ m} < x < 6 \text{ m}$

(14) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$$+\uparrow \Sigma F_y = 0: \quad -8 \text{ kN} + 16 \text{ kN} - 4 \text{ kN} - V = 0$$

$$\circlearrowleft + \Sigma M_x = 0: \quad (8 \text{ kN})x - (16 \text{ kN})(x - 2 \text{ m}) + (4 \text{ kN})(x - 4 \text{ m}) + M = 0$$

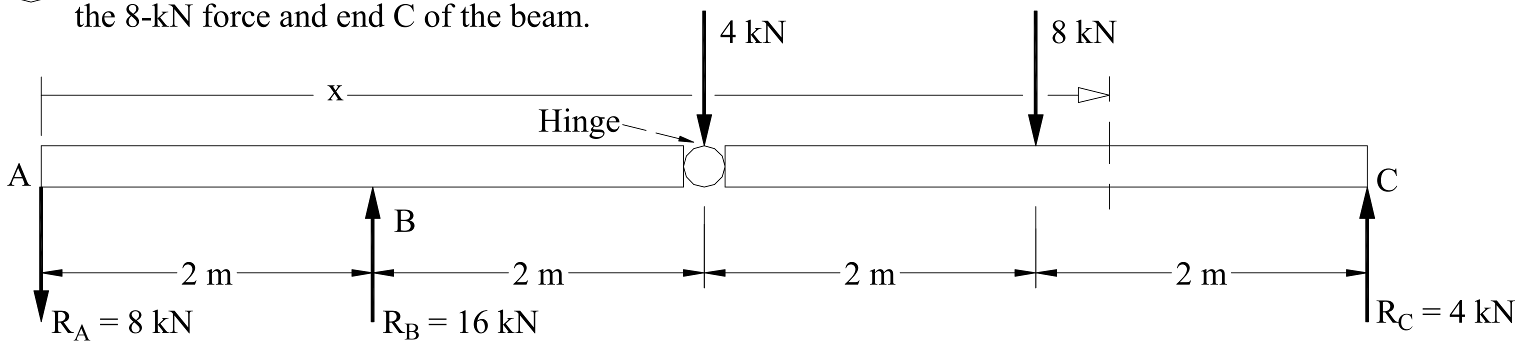
Solving gives

$$V = 4 \text{ kN} \qquad (8)$$

$$M = (4x - 16) \text{ kN}\cdot\text{m} \qquad (9)$$

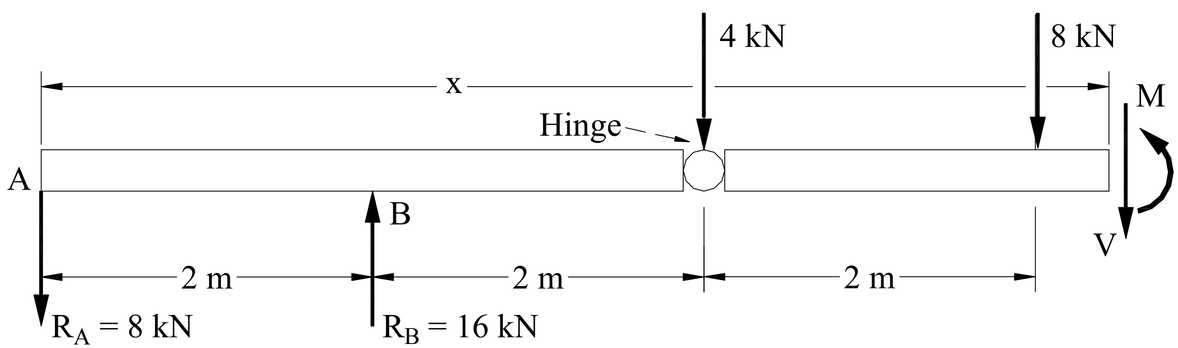
valid for $4 \text{ m} < x < 6 \text{ m}$.

(15) Pass a section through the beam at a point between the 8-kN force and end C of the beam.



6 m < x < 8 m

(16) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow\Sigma F_y = 0$: $-8\text{ kN} + 16\text{ kN} - 4\text{ kN} - 8\text{ kN} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $(8\text{ kN})x - (16\text{ kN})(x - 2\text{ m}) + (4\text{ kN})(x - 4\text{ m}) + (8\text{ kN})(x - 6\text{ m}) + M = 0$

Solving gives

$$V = -4\text{ kN} \tag{10}$$

$$M = (-4x + 32)\text{ kN}\cdot\text{m} \tag{11}$$

valid for 6 m < x < 8 m.

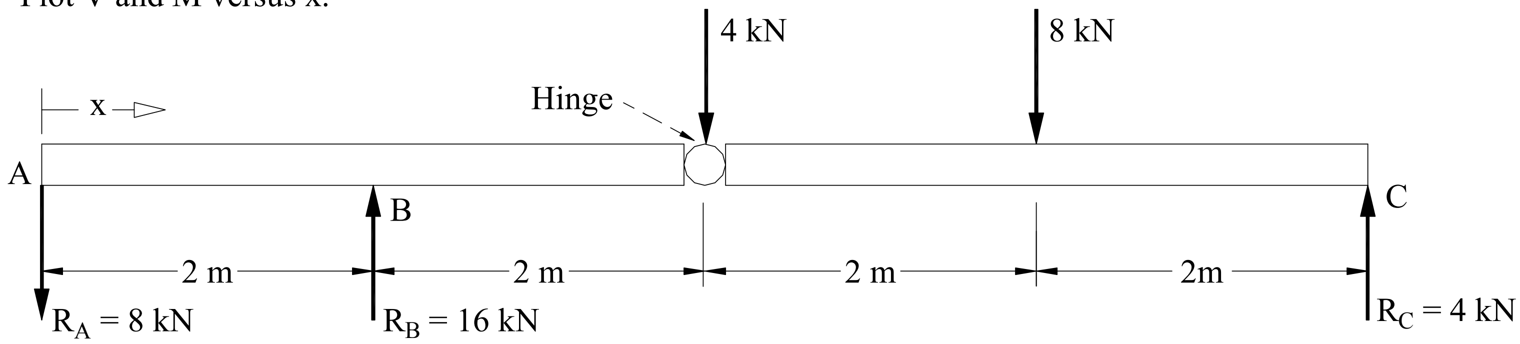
8.2 Shear and Bending-Moment Diagrams: Equation Form Example 6, page 7 of 8

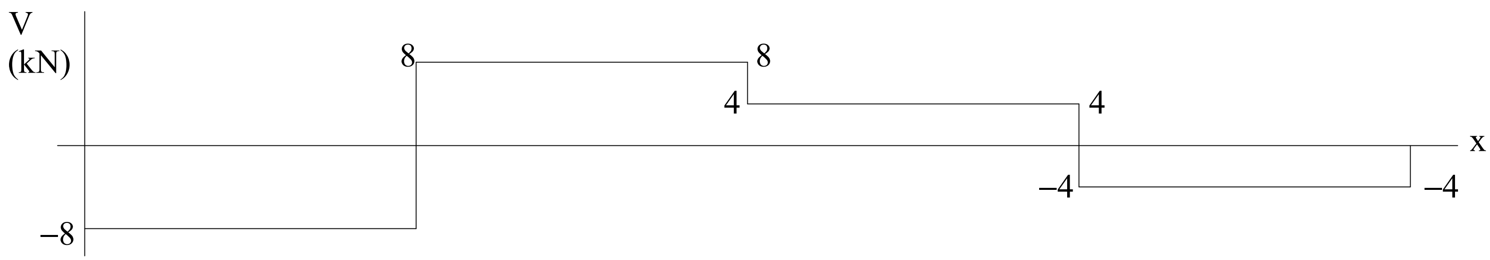
(17) Collect the results from Eqs. 4-11:

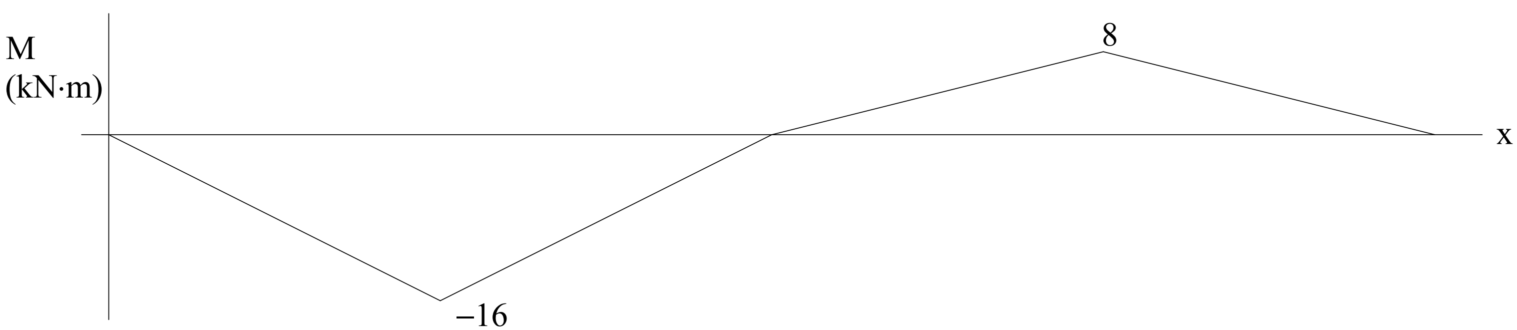
$0 < x < 2$ m	$V = -8$ kN	
	$M = -8x$ kN·m	
2 m $< x < 4$ m	$V = 8$ kN	
	$M = (8x - 32)$ kN·m	←Ans.
4 m $< x < 6$ m	$V = 4$ kN	
	$M = (4x - 16)$ kN·m	
6 m $< x < 8$ m	$V = -4$ kN	
	$M = (-4x + 32)$ kN·m	

8.2 Shear and Bending-Moment Diagrams: Equation Form Example 6, page 8 of 8

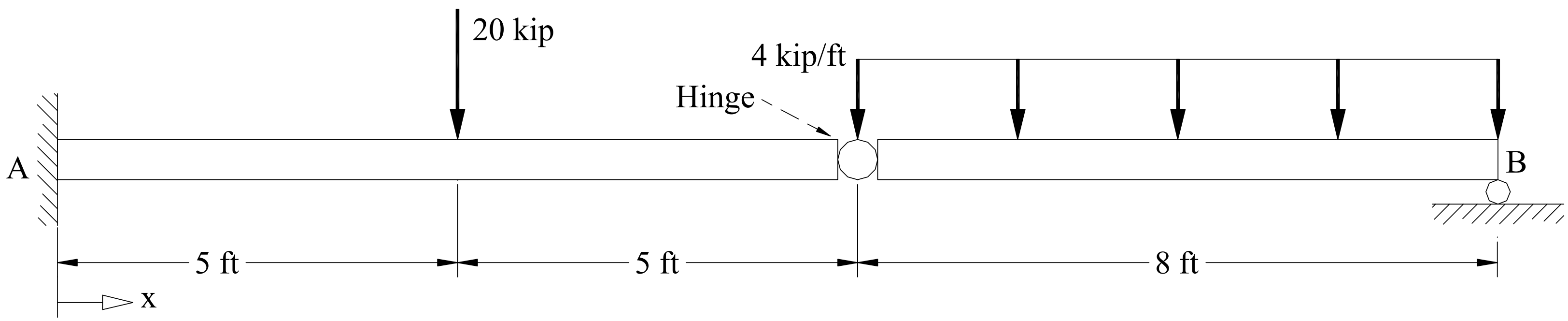
(18) Plot V and M versus x.



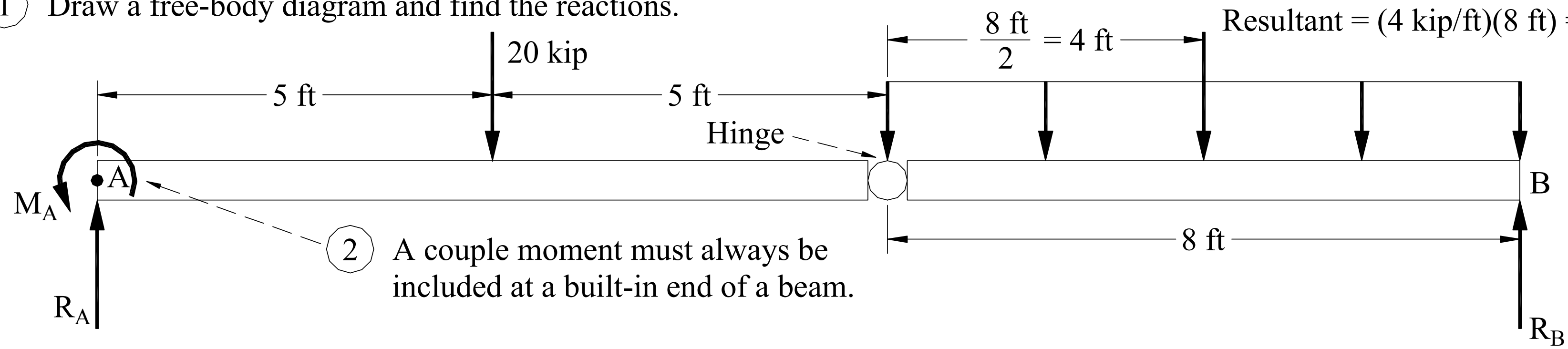




7. Express the shear V and bending moment M as functions of x, the distance from the left end of the beam to an arbitrary point on the beam. Plot V and M vs. x.



(1) Draw a free-body diagram and find the reactions.



$$+\uparrow \Sigma F_y = 0: \; R_A - 20 \text{ kip} - 32 \text{ kip} + R_B = 0 \qquad (1)$$

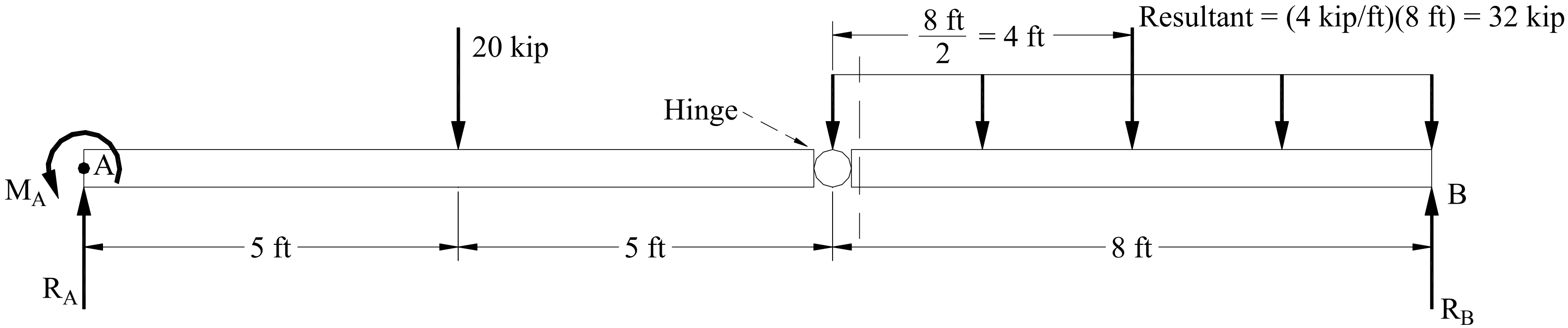
$$\circlearrowleft + \Sigma \overline{M}_A = 0: \; M_A - (20 \text{ kip})(5 \text{ ft}) - (32 \text{ kip})(5 \text{ ft} + 5 \text{ ft} + 4 \text{ ft})$$

$$+ R_B(5 \text{ ft} + 5 \text{ ft} + 8 \text{ ft}) = 0 \qquad (2)$$

(3) Two equations but three unknowns. An additional equation is needed.

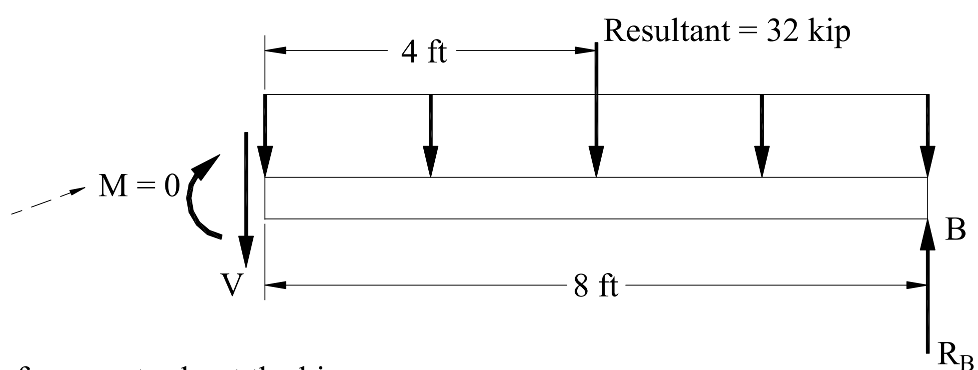
8.2 Shear and Bending-Moment Diagrams: Equation Form Example 7, page 2 of 7

(4) Pass a section through the beam at a point immediately to the right of the hinge.



(5) Draw a free-body diagram of the portion of the beam to the right of the section.

(6) Because the section is next to a hinge, the moment is known to be zero there (that's what we mean by a "hinge").



(7) Write the equilibrium equation for the sum of moments about the hinge.

$$\circlearrowleft + \Sigma M_{hinge} = 0: \; -(32\text{ kip})(4\text{ ft}) + R_B(8\text{ ft}) = 0 \qquad (3)$$

(8) Note that we don't use the equation $\Sigma F_y = 0$, because this equation would introduce an additional unknown, the shear V at the hinge.

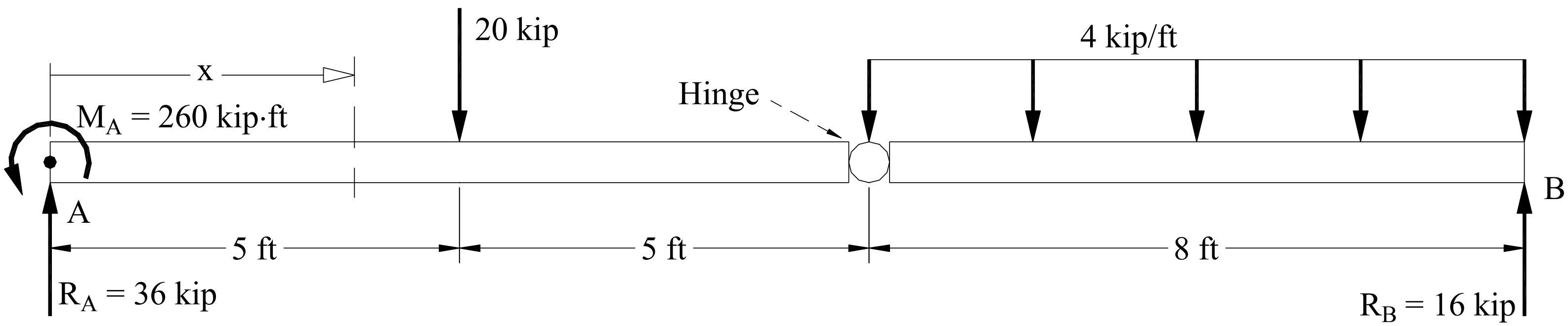
(9) Solving Eqs. 1-3 gives

$R_A = 36$ kip

$R_B = 16$ kip

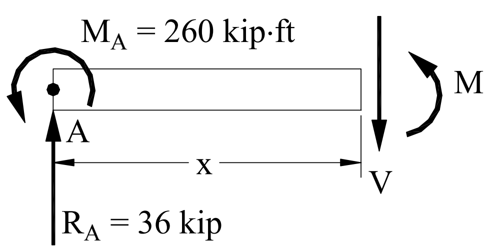
$M_A = 260$ kip·ft

(10) Pass a section through the beam at a point between the left end and the 20-kip force.



$0 < x < 5$ ft

(11) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: 36 kip – V = 0

$\circlearrowleft + \Sigma M_x = 0$: 260 kip·ft – (36 kip)x + M = 0

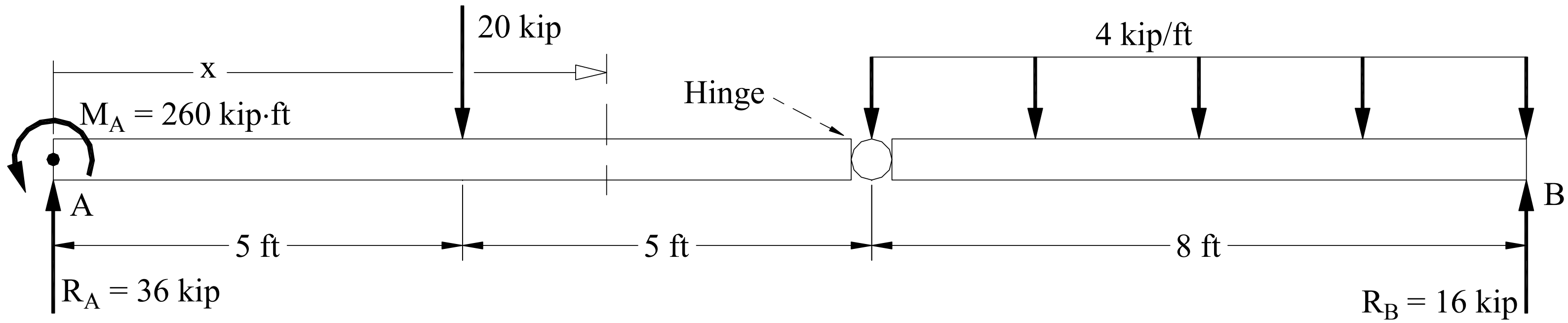
Solving gives

$$V = 36 \text{ kip} \qquad (4)$$

$$M = (36x - 260) \text{ kip·ft} \qquad (5)$$

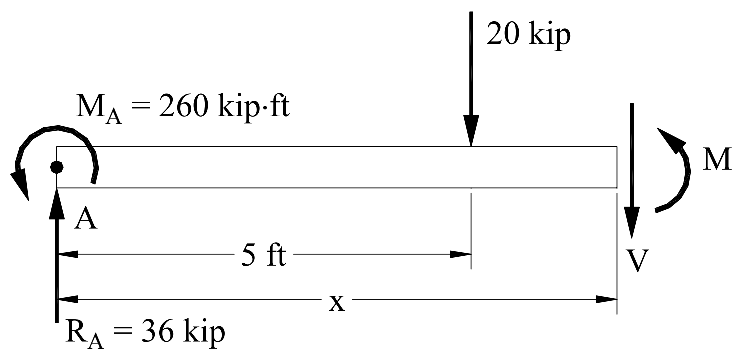
valid for $0 < x < 5$ ft.

(12) Pass a section through the beam at a point between the 20-kip force and the hinge.



5 ft < x < 10 ft

(13) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $36 \text{ kip} - 20 \text{ kip} - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $260 \text{ kip·ft} - (36 \text{ kip})x + 20 \text{ kip}(x - 5 \text{ ft}) + M = 0$

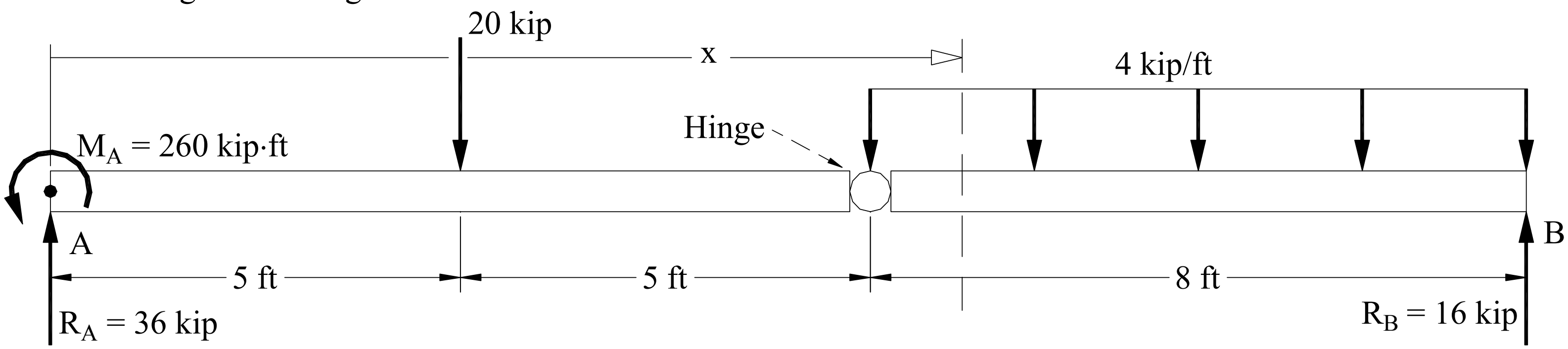
Solving gives

$$V = 16 \text{ kip} \quad (6)$$

$$M = (16x - 160) \text{ kip·ft} \quad (7)$$

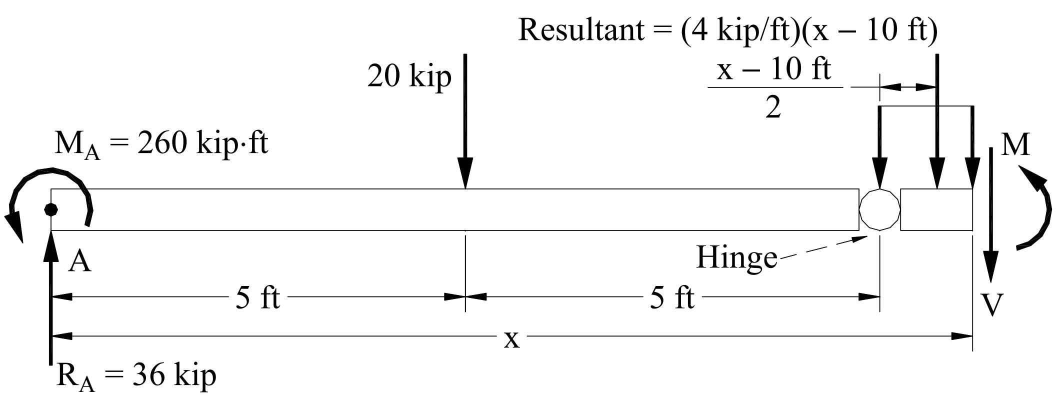
valid for 5 ft < x < 10 ft.

(14) Pass a section through the beam at a point between the hinge and the right end of the beam.



$10 \text{ ft} < x < 18 \text{ ft}$

(15) Draw a free-body diagram of the portion of the beam to the left of the section and find V and M at the section.



$+\uparrow \Sigma F_y = 0$: $36 \text{ kip} - 20 \text{ kip}$

$$- (4 \text{ kip/ft})(x - 10 \text{ ft}) - V = 0$$

$\circlearrowleft + \Sigma M_x = 0$: $260 \text{ kip}\cdot\text{ft} - (36 \text{ kip})x + 20 \text{ kip}(x - 5 \text{ ft})$

$$+ (4 \text{ kip/ft})(x - 10 \text{ ft})\left(\frac{x - 10 \text{ ft}}{2}\right)$$

$$+ M = 0$$

Solving gives

$$V = -4x + 56 \text{ kip} \quad (8)$$

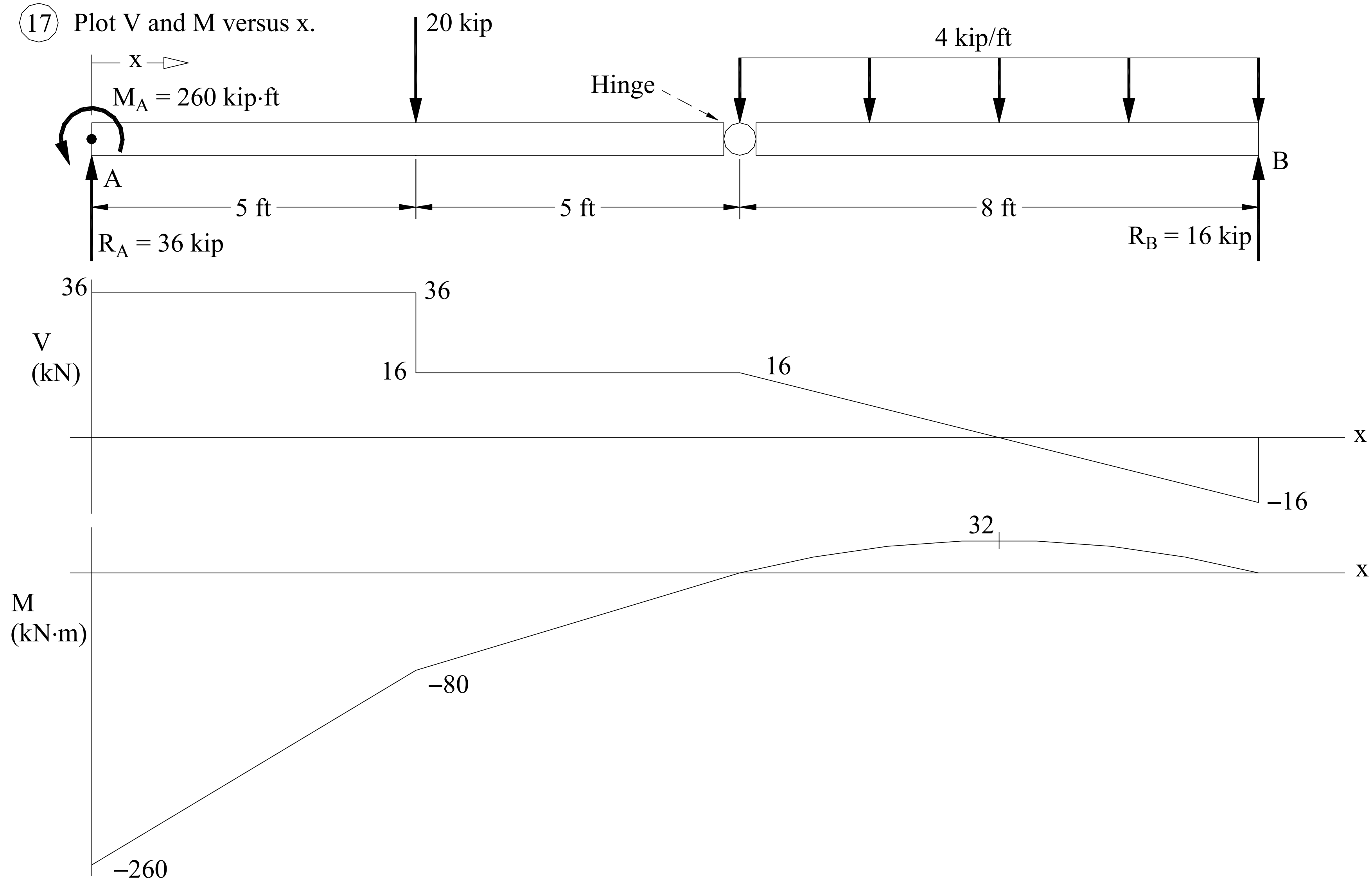
$$M = (-2x^2 + 56x - 360) \text{ kip}\cdot\text{ft} \quad (9)$$

valid for $10 \text{ ft} < x < 18 \text{ ft}$.

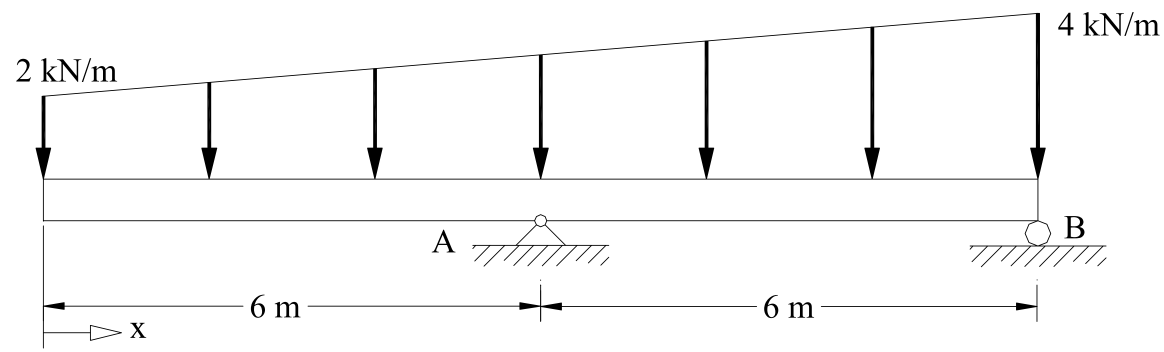
(16) Collect the results from Eqs. 4-9:

$0 < x < 5$ ft	$V = 36$ kip	
	$M = (36x - 260)$ kip·ft	
$5 \text{ ft} < x < 10$ ft	$V = 16$ kip	←Ans.
	$M = (16x - 160)$ kip·ft	
$10 \text{ ft} < x < 18$ ft	$V = (-4x + 56)$ kip	
	$M = (-2x^2 + 56x - 360)$ kip·ft	

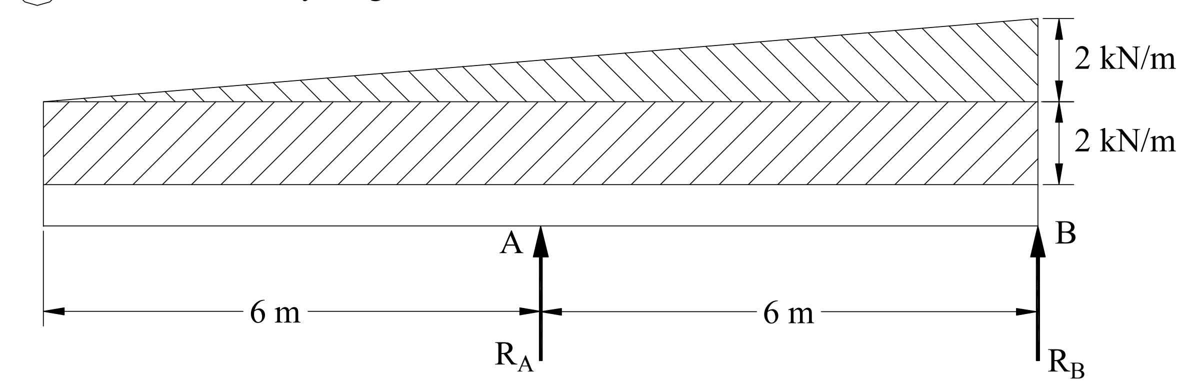
(17) Plot V and M versus x.



8. Express the shear V and bending moment M as functions of x, the distance from the left end of the beam to an arbitrary point on the beam. Plot V and M vs. x.



(1) Draw a free-body diagram and find the reactions.



(2) Replace the trapezoidal distributed load by the sum of a rectangular and triangular load.

(3) Resultant of rectangular load

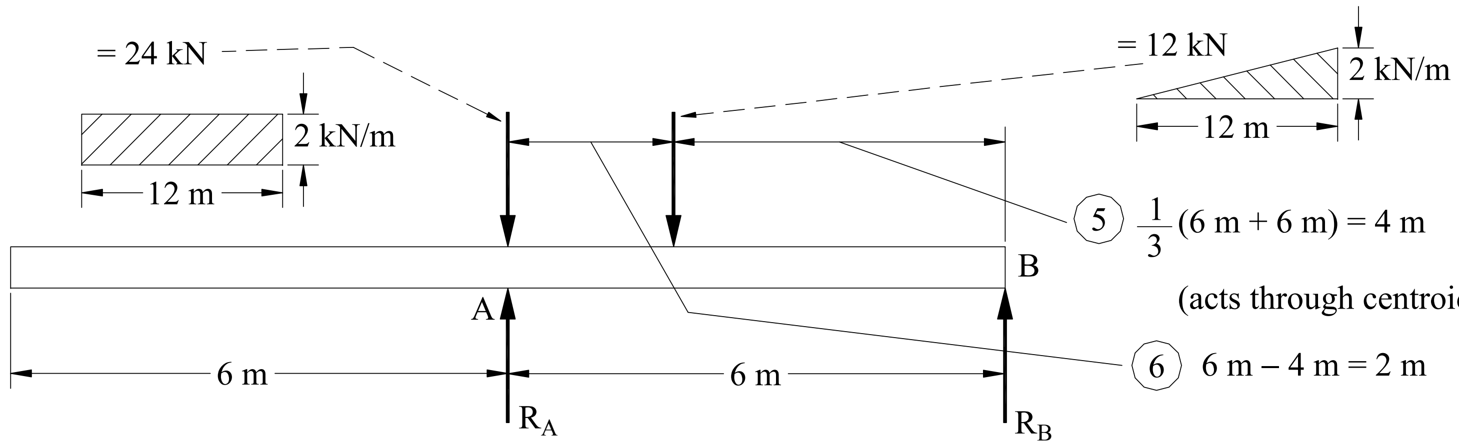
= (12 m)(2 kN/m)

= 24 kN

(4) Resultant of triangular load

$= \frac{1}{2}$ (12 m)(2 kN/m)

= 12 kN



(5) $\frac{1}{3}$(6 m + 6 m) = 4 m

(acts through centroid of triangle)

(6) 6 m – 4 m = 2 m

$+\uparrow \Sigma F_y = 0$: $R_A - 24\text{ kN} - 12\text{ kN} + R_B = 0$

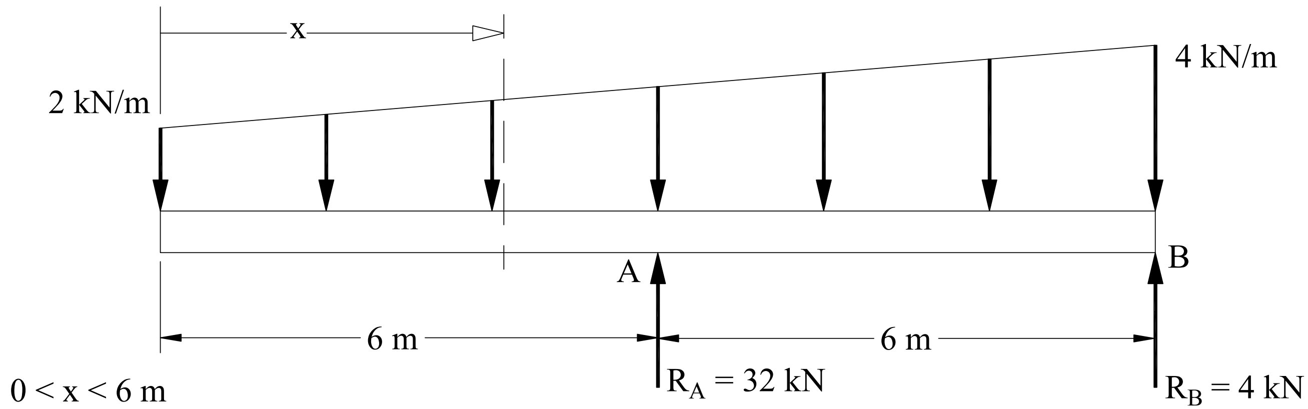
$\circlearrowleft + \Sigma M_A = 0$: $-(12\text{ kN})(2\text{ m}) + R_B(6\text{ m}) = 0$

Solving gives

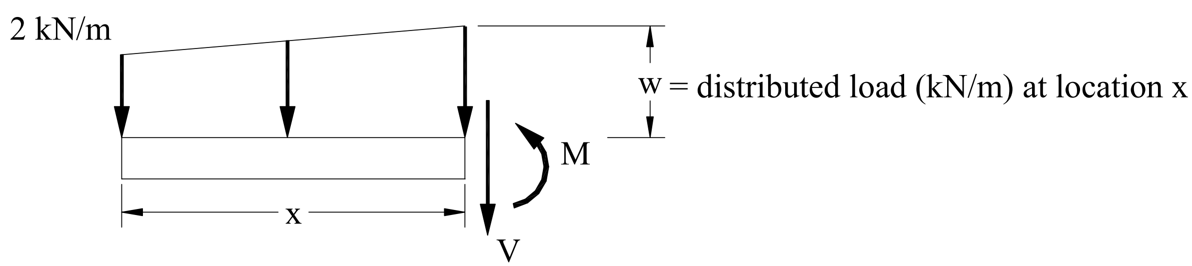
$R_A = 32$ kN

$R_B = 4$ kN

(7) Pass a section through the beam at a point between the left end and the support at A.

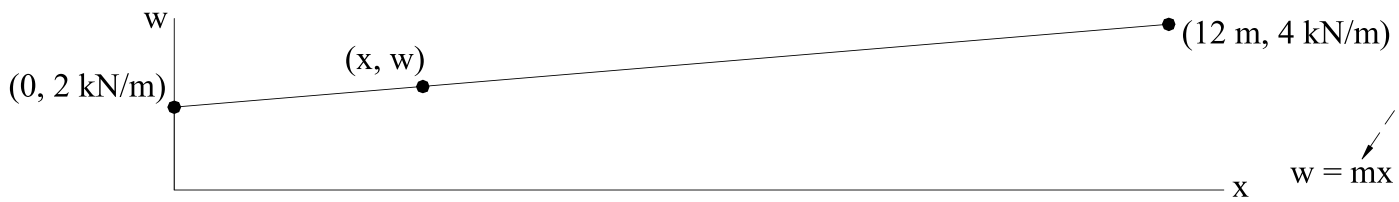


(8) Draw a free-body diagram of the portion of the beam to the left of the section and solve for V and M.



8.2 Shear and Bending-Moment Diagrams: Equation Form Example 8, page 4 of 8

(9) Before we can solve for V and M, we have to express w as a function of x. This can be done by noting that w is a linear function of x and then using the slope-intercept equation for a line.



Slope, Intercept

$$w = mx + b$$

$$= \frac{4\text{ kN/m} - 2\text{ kN/m}}{12\text{ m} - 0}\,x + 2\text{ kN/m}$$

$$= \frac{x}{6} + 2 \qquad (1)$$

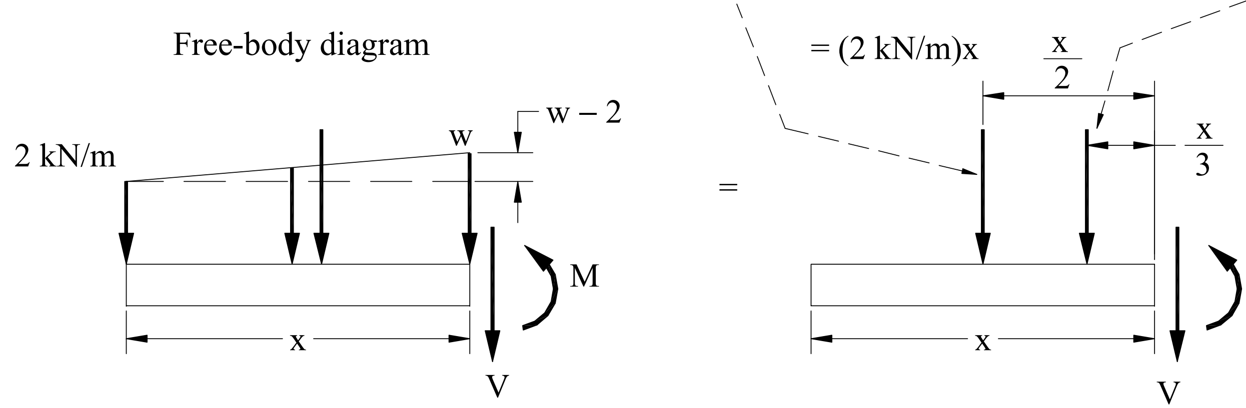
(10) Now the distributed load on the free-body of length x can be replaced by the resultant of a rectangular and triangular load.

(11) Resultant of rectangular load

$= (2\text{ kN/m})x$

(12) Resultant of triangular load

$= \frac{x}{2}(w - 2)$



(13) $+\uparrow \Sigma F_y = 0$: $-(2 \text{ kN/m})x - \frac{x}{2}(w - 2) - V = 0$

$\circlearrowleft + \Sigma M_x = 0$: $(2 \text{ kN/m})(x)(\frac{x}{2}) + [\frac{x}{2}(w - 2)(\frac{x}{3})] + M = 0$

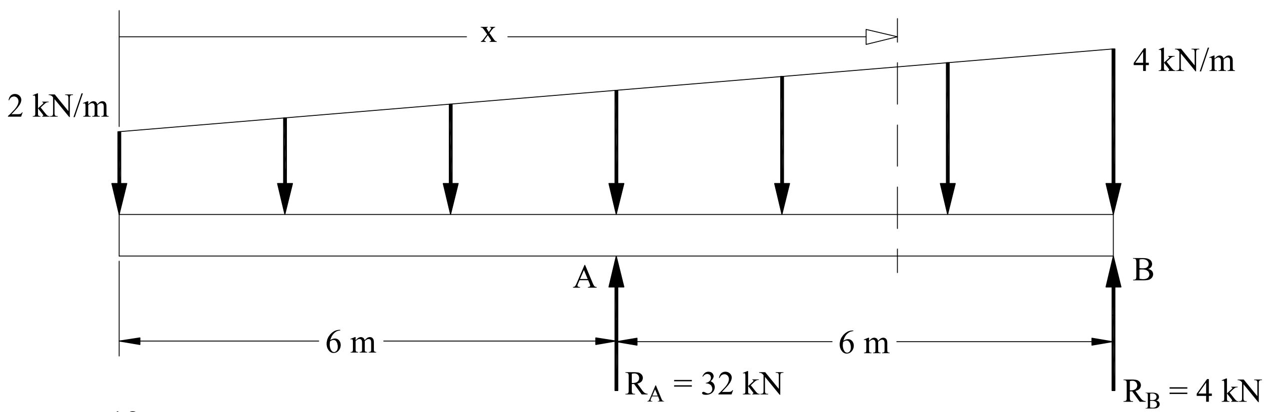
Replacing w in these equations by $w = (x/6) + 2$ from Eq. 1 and solving gives

$$V = \left(-\frac{x^2}{12} - 2x\right) \text{ kN} \qquad (2)$$

$$M = \left(-\frac{x^3}{36} - x^2\right) \text{ kN·m} \qquad (3)$$

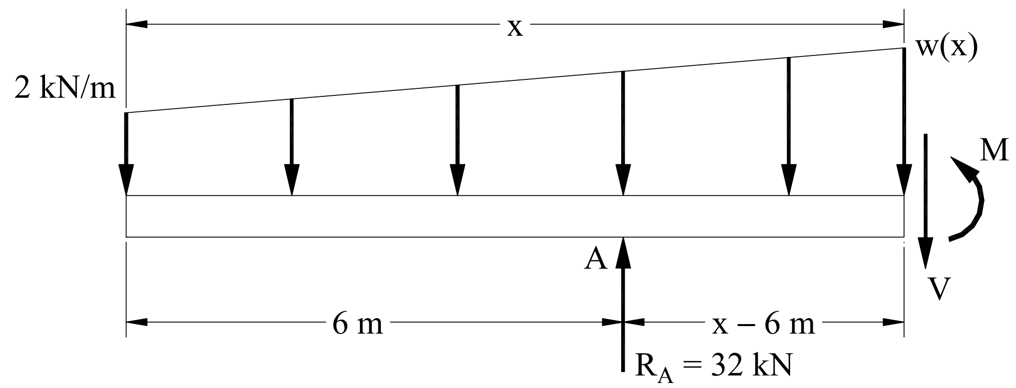
valid for $0 < x < 6$ m.

(14) Pass a section through the beam at a point between the support at A and the support at B.



6 m < x < 12 m

(15) Free-body diagram



$6 \text{ m} < x < 12 \text{ m}$

(16) We can save some work if we note that this free-body diagram is identical to the previous one except that an additional vertical force of 32 kN is present. This increases the shear in Eq. 2 by 32 kN and the moment in Eq. 3 by (32 kN)(x – 6 m) so

$$V = \left(-\frac{x^2}{12} - 2x + 32\right) \text{kN} \qquad (4)$$

$$M = \left(-\frac{x^3}{36} - x^2 + 32x - 192\right) \text{kN·m} \qquad (5)$$

valid for $6 \text{ m} < x < 12 \text{ m}$.

(17) Collect the results from Eqs. 4-11:

$0 < x < 6$ m $\quad V = (-\frac{1}{12}x^2 - 2x)$ kN

$\quad M = (-\frac{1}{36}x^3 - x^2)$ kN·m

6 m $< x < 12$ m $\quad V = (-\frac{1}{12}x^2 - 2x + 32)$ kN

$\quad M = (-\frac{1}{36}x^3 - x^2 + 32x - 192)$ kN·m

←Ans.

(18) Plot V and M versus x.